MCR3U: Functions

Mapping Diagrams and Function Notation

J. Garvin

Mapping Diagrams

A *mapping diagram* is a visual method of connecting elements in the domain of a relation to those in its range.

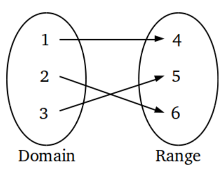
Mapping diagrams are useful when the relation is specified as a set of ordered pairs.

If a mapping diagram maps one element in the domain to more than one element in the range, then the relation is not a function.

Mapping Diagrams

Example

Using the mapping diagram below, state the domain and range of the relation, and determine if it is a function.



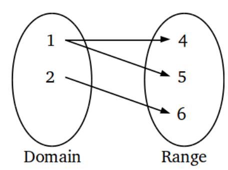
Domain: $= \{1, 2, 3\}$ Range: $= \{4, 5, 6\}$

The relation is a function, since each element in the domain maps to exactly one element in the range.

Mapping Diagrams

Example

Using the mapping diagram below, determine if the relation is a function.



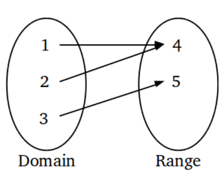
Since the value of 1 in the domain maps to both 4 and 5 in the range, the relation is not a function.

Mapping Diagrams

A function is *one-to-one* if there are no two elements in its domain that map to one element in the range.

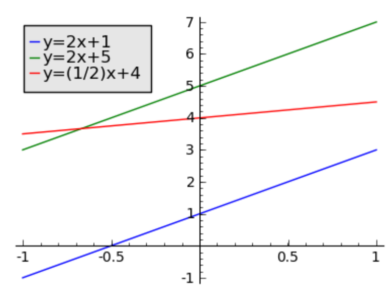
A function is *onto* if all elements in its range are mapped to elements in the domain.

The mapping diagram below illustrates a function that is onto, but not one-to-one.



Function Notation

Suppose you were writing a report, and had data involving the following three linear functions:



Function Notation

While each equation is distinct, it is difficult to refer to an individual function.

For example, how would you indicate that the graph of $y = 2x + 5$ has a y-value of 9 when $x = 2$?

You could refer to as in the previous sentence, but it is wordy.

You could refer to its y-intercept, but what if another function had the same one?

You could mention that it is the green graph, but what if the report is published in black-and-white?

A better solution is to use *function notation* to differentiate each function from another.

Function Notation

Function notation uses a letter as a label, followed by a list of all variables that the function depends on.

For example, the notation $f(x) = 2x + 5$ is read "f of x equals two x plus five", and is equivalent to $y = 2x + 5$.

$f(x)$ is quite common, as are $g(x)$ and $h(x)$, but other variables can be used as well.

- $A(r) = \pi r^2$ describes the area of a circle, based on its radius r.
- $V(e) = e^3$ describes the volume of a cube based on an edge length e.
- $d(t) = 4.9t^2$ describes the distance travelled by a falling object after t seconds.

Function Notation

Returning to the example of the linear relations, it is easy to refer to individual functions if they are labelled $f(x) = 2x + 5$, $g(x) = 2x + 1$ and $h(x) = \frac{1}{2}x + 4$.

To indicate that $y = 2x + 5$ has a y-value of 9 when $x = 2$, we would simply say $f(2) = 9$.

In this case, the x in $f(x)$ has been replaced by 2 to give $f(2)$.

This indicates that anywhere in the function where the variable x appears, we substitute a value of 2 in its place.

Function Notation

Example

Let $f(x) = x^2 - 5x + 1$. Determine $f(0)$, $f(3)$ and $f(-2)$.

$$\begin{aligned} f(0) &= (0)^2 - 5(0) + 1 \\ &= 1 \\ f(3) &= (3)^2 - 5(3) + 1 \\ &= -5 \\ f(-2) &= (-2)^2 - 5(-2) + 1 \\ &= 15 \end{aligned}$$

Function Notation

Example

Let $g(x) = \sqrt{2x - 5}$. Solve for x when $g(x) = 4$.

$$\begin{aligned} g(x) &= \sqrt{2x - 5} \\ 4 &= \sqrt{2x - 5} \\ 16 &= 2x - 5 \\ 21 &= 2x \\ x &= \frac{21}{2} \end{aligned}$$

Function Notation

Example

Let $f(x) = 3x - 1$ and $g(x) = 5x + 3$. Determine the value of $f(x)$ such that $f(x) = g(x)$.

$$\begin{aligned} f(x) &= g(x) \\ 3x - 1 &= 5x + 3 \\ 2x &= -4 \\ x &= -2 \\ f(-2) &= 3(-2) - 1 \\ f(-2) &= -7 \end{aligned}$$

Function Notation

Example

AN EXAMPLE WHERE f(2x+1).

SOLUTION

Function Notation

Example

The volume of copper in a cable with an inner radius of 2 cm depends on the length of the wire. Use function notation to define a function to describe the volume of copper, then determine:

- The volume when the length is 15 cm.
- The length when the volume is 500 cm^3.

The volume of a cylinder, given a radius r and a length l, is $\pi r^2 l$.

Using function notation where V is the volume, l the length and $r = 2$, use $V(l) = 4\pi l$.

Function Notation

To determine the volume for a length of 15 cm, find $V(15)$.

$$\begin{aligned} V(15) &= 4\pi(15) \\ &\approx 188.5\text{cm}^3 \end{aligned}$$

To determine the length for a volume of 500 cm^3, find l when $V(l) = 500$.

$$\begin{aligned} 500 &= 4\pi l \\ l &= \frac{500}{4\pi} \\ &\approx 39.8\text{cm} \end{aligned}$$

Function Notation

A labelling system similar to function notation is *mapping notation*, which uses an arrow in place of an equals sign.

For example, the function $f : x \to x^2 - 4$ is read "function f maps x to $x^2 - 4$" and is equivalent to $f(x) = x^2 - 4$.

We will typically use function notation in this course, but you should be familiar with both systems.

Questions?