MCR3U: Functions

Determining Equations of Sinusoidal Functions

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Equations of Sinusoidal Functions

Recall that a sinusoidal function is a function that can be modelled using a sine wave.

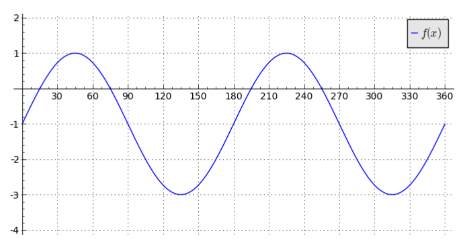
Since $f(x) = \sin x$ and $f(x) = \cos x$ are identical apart from a phase shift, both functions are sinusoidal.

$f(x) = \tan x$ is a trigonometric function, as are $f(x) = \sin x$ and $f(x) = \cos x$, but is *not* sinusoidal, since its graph is completely different.

Since a phase shift is the only difference between sine and cosine, any sinusoidal function that can be modelled using sine can also be modelled using cosine.

Equations of Sinusoidal Functions

Determine an equation for $f(x)$ below.



Equations of Sinusoidal Functions

The axis of the function is at $y = -1$.

The amplitude is 2.

The period is $180°$.

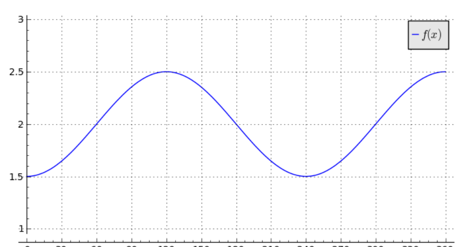
This yields a b-value of $\frac{360}{180} = 2$ in the equation for $f(x)$.

If using sine, there is no phase shift, so a possible equation is $f(x) = 2\sin(2x) - 1$.

If using cosine, there is a phase shift of $45°$, so a possible equation is $f(x) = 2\cos(2(x - 45)) - 1$.

Equations of Sinusoidal Functions

Determine an equation for $f(x)$ below.



Equations of Sinusoidal Functions

The axis of the function is at $y = 2$.

The amplitude is $\frac{1}{2}$.

The period is $240°$.

This yields a b-value of $\frac{360}{240} = \frac{3}{2}$ in the equation for $f(x)$.

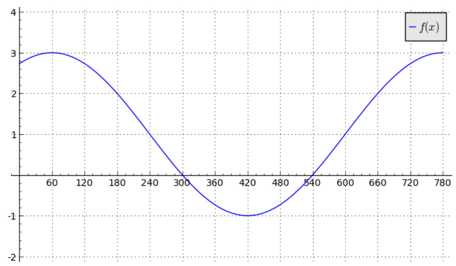
If using sine, there is a phase shift of $60°$, so a possible equation is $f(x) = \frac{1}{2}\sin\left(\frac{3}{2}(x - 60)\right) + 2$.

If using cosine, there is a phase shift of $120°$, so a possible equation is $f(x) = \frac{1}{2}\cos\left(\frac{3}{2}(x - 120)\right) + 2$.

An even better option is to use "negative cosine" (reflected in the x-axis), which has no phase shift. A possible equation is $f(x) = -\frac{1}{2}\cos\left(\frac{3}{2}x\right) + 2$.

Equations of Sinusoidal Functions

Determine an equation for $f(x)$ below.



Equations of Sinusoidal Functions

The axis of the function is at $y = 1$.

The amplitude is 2.

The period is 720°.

This yields a b-value of $\frac{360}{720} = \frac{1}{2}$ in the equation for $f(x)$.

If using sine, there is a phase shift of 600°, so a possible equation is $f(x) = 2\sin\left(\frac{1}{2}(x - 600)\right) + 1$.

If using cosine, there is a phase shift of 60°, so a possible equation is $f(x) = 2\cos\left(\frac{1}{2}(x - 60)\right) + 1$.

Using either sine or cosine, a phase shift is required.

Equations of Sinusoidal Functions

Example

Determine an equation of a sinusoidal function that satisfies the following properties:

- The first minimum, after 0°, occurs at $x = 45°$.
- The first maximum, after 0°, occurs at 135° after the minimum.
- The minimum and maximum values of the function are 4 and 12 respectively.

Equations of Sinusoidal Functions

The axis of the function is at $y = \frac{4+12}{2} = 8$.

The amplitude of the function is $\left|\frac{4-12}{2}\right| = 4$.

Since the minimum and maximum oddur 135° apart, the period of the function is double this value, or 270°.

This yields a b-value of $\frac{360}{270} = \frac{4}{3}$ in the equation.

Since the first maximum occurs at $45 + 135 = 180°$, we can use cosine wth this phase shift.

Thus, a possible equation for the function is $f(x) = 4\cos\left(\frac{4}{3}(x - 180)\right) + 8$.

Questions?