MCR3U: Functions

Compound Interest

Part 2: Present Value

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Compound Interest

Recap

$5 800 is invested into an account paying 4.6%/a interest, compounded bi-monthly. What is the value of the investment after 8 years?

Use $P = \$5\,800$, $i = 0.046$, $n = 8$, and $c = 6$.

$$\begin{aligned} FV &= \$5\,800\left(1+\tfrac{0.046}{6}\right)^{8\times 6} \\ &\approx \$8\,368.33 \end{aligned}$$

Compound Interest

Earlier questions dealt with the future value of investments, or with parameters (e.g. time, interest rate) involved with these values.

In some cases, however, we are interested in the *present value* of an investment – how much principal is required to grow to a specific amount.

For example, we may want to invest some money into an account so that it is worth $5 000 in 10 years.

Since the money will earn compound interest during those 10 years, we should be able to invest less than $5 000 and let it grow to the desired amount.

Compound Interest

To obtain a formula for the present value of an investment, we simply need to isolate P in the future value equation.

Present Value of an Investment, Compounded Regularly

The amount of money, P, needed to be invested in an account at i%/a for n years, with a compounding frequency of c times per year, to obtain a future value of FV is given by $P = \dfrac{FV}{\left(1+\frac{i}{c}\right)^{nc}}$, or $P = FV\left(1+\frac{i}{c}\right)^{-nc}$.

It is also possible to use the future value equation and rearrange after simplification.

Compound Interest

Example

How much money needs to be invested into an account paying 3.5%/a interest, compounded monthly, to grow to $5 000 in 10 years?

Use $FV = \$5\,000$, $i = 0.035$, $n = 10$ and $c = 12$.

$$\begin{aligned} P &= \frac{5\,000}{\left(1+\frac{0.035}{12}\right)^{10\times 12}} \\ &\approx \$3\,525.24 \end{aligned}$$

It takes a little more than $3 500 to grow to $5 000 in 10 years, due to the interest earned on the amount.

Compound Interest

Example

It is estimated that first-year tuition at a university will cost $8 000. Which is the better option?

- Investing at 3.8%/a interest, compounded bi-monthly, for 5 years, or
- Investing at 2.1%/a interest, compounded bi-monthly, for 10 years?

Determine the present value of each investment, to determine how much needs to be invested to grow to $8 000.

$$\begin{aligned} P &= \frac{8\,000}{\left(1+\frac{0.038}{6}\right)^{6\times 5}} \\ &\approx \$6\,619.64 \end{aligned} \qquad \begin{aligned} P &= \frac{8\,000}{\left(1+\frac{0.021}{6}\right)^{6\times 10}} \\ &\approx \$6\,487.05 \end{aligned}$$

The second option is better, with a lower initial investment.

Compound Interest

Example

A young entrepreneur applies for a loan at a bank. According to her business plan, she estimates that she will be able to pay back \$20 000 in 3 years. If the bank charges 4.5%/a interest, compounded quarterly, how much can she borrow?

The future value of the loan is \$20 000, and the amount of the loan is its present value.

$$P = \frac{20\,000}{\left(1 + \frac{0.045}{4}\right)^{3\times 4}}$$
$$\approx \$17\,487.49$$

She can borrow just under \$17 500 from the bank.

Questions?