

Change of Base

J. Garvin



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Change of Base

Recap

Simplify $(8x)^{\frac{3}{2}} \left(x^{\frac{5}{2}}\right)$ as a single power.

$$\begin{aligned}(8x)^{\frac{3}{2}} \left(x^{\frac{5}{2}}\right) &= 8^{\frac{3}{2}} x^{\frac{3}{2}} x^{\frac{5}{2}} \\ &= (\sqrt{8})^3 x^{\frac{3}{2} + \frac{5}{2}} \\ &= 8\sqrt{8} x^{\frac{8}{2}} \\ &= 16\sqrt{2} x^4\end{aligned}$$

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Recall that the exponent laws for multiplication and division only apply if the bases are the same.

For this reason, we can simplify $2^5 \times 2^3$ as 2^8 , but we cannot simplify $2^5 \times 3^2$ as 6^7 , or $2 \cdot 5^7$, etc.

It is often possible, however, to represent powers using different bases.

For example, $9^2 = 81$, and $9 = 3^2$, so $9^2 = (3^2)^2 = 3^4 = 81$.

By changing bases, it may be possible to simplify expressions that involve powers with different bases.

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Example

Rewrite 8^x as a power of 2.

Since $8 = 2^3$, $8^x = (2^3)^x = 2^{3x}$.

Example

Rewrite 64^{3x+1} as a power of 4.

Since $64 = 4^3$, $64^{3x+1} = (4^3)^{3x+1} = 4^{9x+3}$.

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Example

Simplify $3^{5x} \cdot 9^{x+3}$.

$$\begin{aligned}3^{5x} \cdot 9^{x+3} &= 3^{5x} \cdot (3^2)^{x+3} \\ &= 3^{5x} \cdot 3^{2x+6} \\ &= 3^{7x+6}\end{aligned}$$

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Example

Simplify $\frac{32^{4x-3}}{8^{2x+5}}$.

$$\begin{aligned}\frac{32^{4x-3}}{8^{2x+5}} &= \frac{(2^5)^{4x-3}}{(2^3)^{2x+5}} \\ &= \frac{2^{20x-15}}{2^{6x+15}} \\ &= 2^{14x-30}\end{aligned}$$

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Example

Simplify $\frac{2^{4x+1} \cdot 8^{x-5}}{16^{2x+3}}$.

$$\begin{aligned} \frac{2^{4x+1} \cdot 8^{x-5}}{16^{2x+3}} &= \frac{2^{4x+1} \cdot (2^3)^{x-5}}{(2^4)^{2x+3}} \\ &= \frac{2^{4x+1} \cdot 2^{3x-15}}{2^{8x+12}} \\ &= \frac{2^{7x-14}}{2^{8x+12}} \\ &= 2^{-x-26} \text{ or } \frac{1}{2^{x+26}} \end{aligned}$$

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Example

Simplify $0.2 \cdot \frac{125^{3-2x}}{25^{4x-1}}$.

$$\begin{aligned} 0.2 \cdot \frac{125^{3-2x}}{25^{4x-1}} &= \frac{1}{5} \cdot \frac{(5^3)^{3-2x}}{(5^2)^{4x-1}} \\ &= \frac{5^{9-6x}}{5^1 \cdot 5^{8x-2}} \\ &= \frac{5^{9-6x}}{5^{8x-1}} \\ &= 5^{-14x+10} \text{ or } \frac{1}{5^{14x-10}} \end{aligned}$$

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Questions?

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