MCR3U: Functions

Arithmetic Sequences

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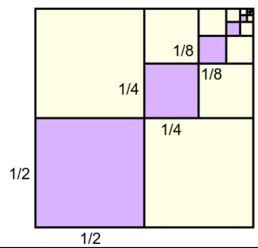

Arithmetic Sequences

A *sequence* is an ordered set of values.

For example, the numbers $3, 10, 17, 24, \dots$ form a sequence. In this case, the ellipsis indicates that the sequence has infinitely many terms.

Terms are often references by their position, or *index*. In this case, $t_1 = 3$, $t_2 = 10$, $t_3 = 17$, etc. t_1 is the *initial term*, while t_n is the nth term.

Some sequences have predictable patterns. In the sequence above, each term is 7 greater than the previous term.

Arithmetic Sequences

If a sequence has a *common difference* between any two successive terms, it is called an *arithmetic sequence*.

Arithmetic Sequences

Example

Determine whether the sequence beginning $6, 23, 40, 57, \dots$ is arithmetic or not.

Since there is a common difference of 17 between each term (e.g. $23 - 6 = 17$), the sequence is arithmetic.

Example

Determine whether the sequence beginning $19, 28, 35, 40, \dots$ is arithmetic or not.

Since $28 - 19 = 9$, but $35 - 28 = 7$, there is no common difference. Therefore, the sequence is not arithmetic.

Arithmetic Sequences

If a sequence is arithmetic, it is possible to use the common difference to extend the sequence, or to determine the values of subsequent terms.

Example

Determine the next three terms of the arithmetic sequence beginning $8, 15, 22, \dots$.

The common difference is $15 - 8 = 7$. Thus, the next three terms are $t_4 = 22 + 7 = 29$, $t_5 = 29 + 7 = 36$ and $t_6 = 36 + 7 = 43$.

While this method works well for small values of n, it would be tedious to use the common difference alone to determine the value of the 1 000th term.

Arithmetic Sequences

An alternative to listing the first few terms of an arithmetic sequence is to state an equation for the *general term* – a formula that can be used to calculate the value of any specific term.

Consider the sequence $t_1, t_2, t_3, \dots, t_{n-1}, t_n$. Since the sequence is arithmetic, there is a common difference between each term.

Thus, $t_2 = t_1 + d$, $t_3 = t_2 + d = (t_1 + d) + d = t_1 + 2d$, etc.

Therefore, the arithmetic sequence can be rewritten as $t_1, t_1 + d, t_1 + 2d, \dots, t_1 + (n-2)d, t_1 + (n-1)d$.

Arithmetic Sequences

This gives us a formula for the value of the nth term.

General Term of an Arithmetic Sequence

The general term of an arithmetic sequence is given by $t_n = t_1 + (n-1)d$, where t_n is the value of the nth term, t_1 is the value of the 1st term, and d is the common difference between successive terms.

Since an arithmetic sequence represents linear growth, the formula for the general term resembles that for a linear relation when expanded and simplified.

Arithmetic Sequences

Example

Develop a formula for the general term of the arithmetic sequence that begins $82, 71, 60, \ldots$, then determine the value of the 20th term.

$t_1 = 82$ and $d = 71 - 82 = -11$. Therefore, an equation is:

$$t_n = 82 + (n-1)(-11)$$
$$t_n = 82 - 11n + 11$$
$$t_n = 93 - 11n$$

Using $n = 20$, $t_{20} = 93 - 11(20) = -127$.

Arithmetic Sequences

Example

In an arithmetic sequence, the 3rd term is -98 and the 9th term is 76. Determine the value of the 100th term.

To go from the 3rd term to the 9th term, it is necessary to add the common difference six times.

$$-98 + 6d = 76$$
$$6d = 174$$
$$d = 29$$

Now that the value of the common difference is known, solve for t_1 using the formula for the general term.

$$-98 = t_1 + (3-1)(29)$$
$$t_1 = -156$$

Arithmetic Sequences

Develop a formula for the general term.

$$t_n = -156 + (n-1)(29)$$
$$t_n = -156 + 29n - 29$$
$$t_n = -185 + 29n$$

Finally, substitute $n = 100$ to find the value of the 100th term.

$$t_{100} = -185 + 29(100)$$
$$= 2\,715$$

Arithmetic Sequences

Example

Determine the number of terms in the arithmetic sequence $15, 38, \ldots, 613$.

We know that $t_1 = 15$ and that $t_n = 613$, for some n.

There is a constant difference of $38 - 15 = 23$. Substitute these three values into the formula for the general term.

$$613 = 15 + (n-1)(23)$$
$$598 = (n-1)(23)$$
$$26 = n - 1$$
$$n = 27$$

Therefore, the sequence has 27 terms.

Arithmetic Sequences

Example

A seating section in a concert hall has 18 seats in the first row. Each subsequent row has 2 additional seats than the row before it. How many seats are in the 25th row?

Substitute $t_1 = 18$, $d = 2$ and $n = 25$ into the equation for the general term, and solve for t_{25}.

$$t_{25} = 18 + (25-1)(2)$$
$$= 66$$

There are 66 seats in the 25th row.

Questions?