MCR3U: Functions

Solving Quadratic Equations

Part 2: Applications

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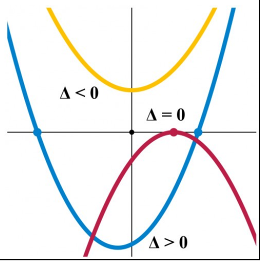


Solving Quadratic Equations

Many applications involving trying to determine the zeroes of a quadratic equation.

Quadratic equations can be solved by factoring, completing the square, or using the quadratic formula.

Some methods are easier than others, but you may use any method which is appropriate.

Number Questions

Example

Determine two numbers that have a sum of 45 and a product of 470.

Let x and y be the two numbers.

$$x + y = 45$$
$$y = 45 - x$$
$$xy = 470$$
$$x(45 - x) = 470$$
$$x^2 - 45x + 470 = 0$$
$$(x - 17)(x - 28) = 0$$

The two numbers are 17 and 28.

Projectile Questions

Example

A ball is kicked into the air with a velocity of 30 m/s. Its height, in metres, after t seconds is given by $h(t) = -4.9t^2 + 30t + 5$. Determine when the ball hits the ground, to the nearest tenth of a second.

$$t = \frac{-30 \pm \sqrt{30^2 - 4(-4.9)(5)}}{2(-4.9)}$$
$$t = \frac{-30 \pm \sqrt{998}}{-9.8}$$
$$t \approx 6.3 \text{ or } -0.2$$

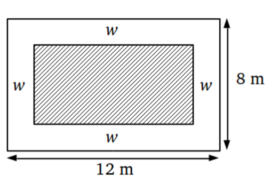
Since it does not make sense to have a negative time in this scenario, the only solution is approximately 6.3 seconds.

Geometry Questions

Example

A walkway of uniform width is to be built inside the edge of a garden measuring 8 metres by 12 metres. The new area of the garden must be 75% of its original area. Determine the *exact* width of the walkway.

A diagram helps.



Geometry Questions

The original area of the garden is $8 \times 12 = 96$ m^2, while the new area is $96 \times 0.75 = 72$ m^2.

Let $A(w)$ be the new area of the garden based on the width, w, of the walkway.

$$A(w) = (8 - 2w)(12 - 2w)$$
$$72 = 96 - 40w + 4w^2$$
$$4w^2 - 40w + 24 = 0$$
$$w^2 - 10w + 6 = 0$$

Geometry Questions

$$w = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(6)}}{2(1)}$$
$$w = \frac{10 \pm \sqrt{76}}{2}$$
$$w = \frac{10 \pm 2\sqrt{19}}{2}$$
$$w = 5 \pm \sqrt{19}$$

Since $5 + \sqrt{19} \approx 9.4$, this width is not possible. The only solution is $5 - \sqrt{19}$ m, which is approximately 64 cm.

Cost/Revenue/Profit Questions

Example

A hot dog vendor sells, on average, 1200 hot dogs per week at \$2.50 each. For each \$0.10 increase in price, he sells 100 fewer hot dogs.

- How much must he charge per hot dog to bring in some amount of revenue?
- What price, to the nearest dime, results in a weekly revenue of at least \$2 000?

Cost/Revenue/Profit Questions

Let $R(n)$ be the revenue function, based on the number, n, of price increases.

$$R(n) = (2.5 + 0.1n)(1\,200 - 100n)$$
$$0 = (2.5 + 0.1n)(1\,200 - 100n)$$
$$n = -25 \text{ or } 12$$

The vendor will earn some revenue if he charges more than $(2.5 + 0.1(-25)) = \$0$, but less than $(2.5 + 0.1(12)) = \$3.70$.

Cost/Revenue/Profit Questions

To determine the prices for a weekly revenue of \$2 000, substitute $R(n) = 2\,000$.

$$2\,000 = (2.5 + 0.1n)(1\,200 - 100n)$$
$$2\,000 = 3\,000 - 130n - 10n^2$$
$$n^2 + 13n - 100 = 0$$
$$n = \frac{-13 \pm \sqrt{13^2 - 4(1)(-100)}}{2(1)}$$
$$n \approx -18.4 \text{ or } 5.4$$

To earn at least \$2 000 revenue per week, the vendor must charge between $(2.5 + 0.1(-18)) = \$0.70$ and $(2.5 + 0.1(5)) = \$3$, for \$2 100 or greater per week.

Questions?