MCR3U: Functions

Applications of Trigonometric Functions

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Applications of Trigonometric Functions

Example

A ferris wheel has a diameter of 16 m, with a minimum height of 2 m above the ground. It takes 20 seconds for the ferris wheel to complete one rotation. If a rider boards a car when it is at its lowest point, determine an equation to model the rider's height after t seconds, and sketch a graph of two full revolutions.

Since the diameter of the ferris wheel is 16 m, its radius is 8 m and corresponds to the amplitude of the function.

The axis is at $y = 2 + 8 = 10$, since the lowest point is 2 m above the ground.

The period is 20 seconds, so a b-value for the function is $b = \frac{360}{20} = 18$.

Applications of Trigonometric Functions

Since a rider boards at the lowest point, using a reflection of cosine in the x-axis will eliminate a phase shift.

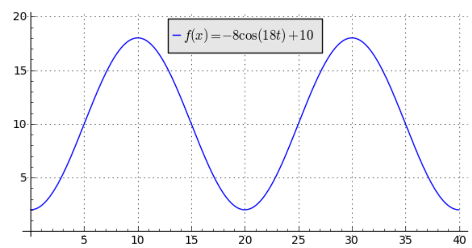
A possible equation is $h(t) = -8\cos(18t) + 10$, where h is the rider's height in metres, and t is the time in seconds.

An alternate equation is $h(t) = 8\cos(18(t-10)) + 10$, since the first maximum will occur after 10 seconds.

Another possible equation is $h(t) = 8\sin(18(t-5)) + 10$, since the first point at which the ferris wheel will be level with the axis, moving upward, will occur 5 seconds into the ride.

Applications of Trigonometric Functions

A sketch of two rotations is below.



Applications of Trigonometric Functions

Example

Determine the height of a rider in the previous example after 12 seconds.

Substitute $t = 12$ into the equation.

$$\begin{aligned} h(12) &= -8\cos(18(12)) + 10 \\ &= -8\cos(216) + 10 \\ &\approx 16.47 \end{aligned}$$

The rider is approximately 16.5 m above the ground after 12 seconds.

Applications of Trigonometric Functions

Example

Determine the first time at which a rider is 14 m above the ground.

Substitute $h(t) = 14$ into the equation.

$$\begin{aligned} 14 &= -8\cos(18t) + 10 \\ 4 &= -8\cos(18t) \\ -\tfrac{1}{2} &= \cos(18t) \\ 18t &= \cos^{-1}\left(-\tfrac{1}{2}\right) \\ 18t &= 120 \\ t &= \tfrac{20}{3} \end{aligned}$$

The rider is first 14 m above the ground after approximately 6.7 seconds.

Applications of Trigonometric Functions

Example

Determine the length of time, during one rotation, in which a rider is below 7 m above the ground.

Substitute $h(t) = 7$ into the equation.

$$\begin{aligned} 7 &= -8\cos(18t) + 10 \\ -3 &= -8\cos(18t) \\ \tfrac{3}{8} &= \cos(18t) \\ 18t &= \cos^{-1}\left(\tfrac{3}{8}\right) \\ 18t &\approx 68 \\ t &\approx 3.78 \end{aligned}$$

Applications of Trigonometric Functions

The rider first reaches a height of 7 m after approximately 3.78 seconds.

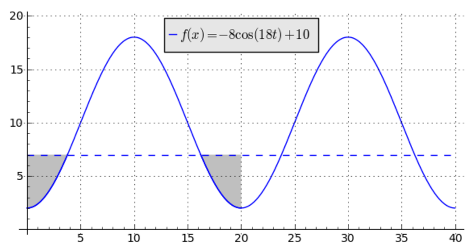
This occurs when the ferris wheel is moving upward, from its lowest point.

By symmetry, a rider will also be below 7 m on the descent.

Therefore, the total time is approximately 7.56 seconds.

Applications of Trigonometric Functions

The graph below indicates the times during which the rider is below 7 m.



Applications of Trigonometric Functions

Example

A flagpole waves back and forth in a strong wind, which pushes it up to 5 cm (left, then right) from its rest position. If the flagpole moves from left to right (or vice versa) 8 times per second, determine an equation that models the horizontal distance from rest position after t seconds, and sketch a graph of two full cycles.

Since the pole moves 5 cm to either side, the amplitude is 5.

The axis of the function is at $y = 0$.

Since the pole moves from one extreme to the other 8 times per second, it completes 4 full cycles per second.

Therefore, it takes $\frac{1}{4}$ second to complete one full cycle.

Applications of Trigonometric Functions

A b-value for the function, then, is $b = \frac{360}{\frac{1}{4}} = 4 \times 360 = 1440$.

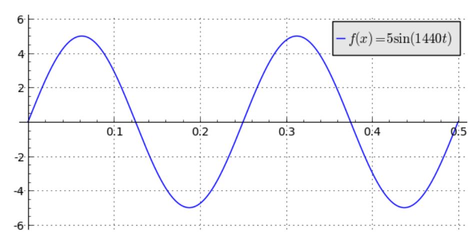
Since we are measuring the distance from rest, use sine to avoid a phase shift.

A function that models the flagpole's horizontal distance is $d(t) = 5\sin(1440t)$.

Since the flagpole first moves left, define left as positive (to match sine) and right as negative.

Applications of Trigonometric Functions

A sketch of two cycles is below.



Applications of Trigonometric Functions

Example

Determine the position of the flagpole after two tenths of a second.

Substitute $t = 0.2$ into the equation.

$$\begin{aligned} d(0.2) &= 5\sin(1440(0.2)) \\ &= 5\sin(288) \\ &\approx -4.76 \end{aligned}$$

The pole is approximately 4.76 cm to the right.

Applications of Trigonometric Functions

Example

Determine the time at which the flagpole is 3 cm to the left, moving toward rest position.

Substitute $d(t) = 3$ into the equation.

$$\begin{aligned} 3 &= 5\sin(1440t) \\ \tfrac{3}{5} &= \sin(1440t) \\ 1440t &= \sin^{-1}\left(\tfrac{3}{5}\right) \\ 1440t &\approx 36.9 \\ t &\approx 0.0256 \end{aligned}$$

Applications of Trigonometric Functions

The first time the flagpole is 3 cm to the left is approximately 0.0256 seconds.

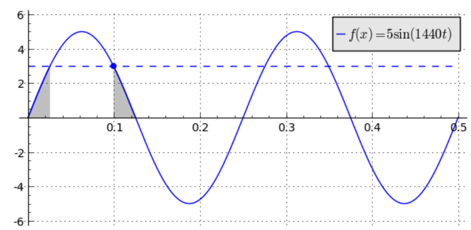
At this time, however, the pole is moving left, *away* from rest position.

Since it takes $\frac{1}{4}$ second to complete one cycle, it takes $\frac{1}{8}$ second to complete one half-cycle.

Thus, the time at which the pole is left of rest position, moving toward it, is approximately $0.125 - 0.0256 \approx 0.0994$ seconds.

Applications of Trigonometric Functions

A graph of the scenario is below. Remember that we defined left as positive.



Questions?