EXPONENTIAL FUNCTIONS	EXPONENTIAL FUN
	Applications of Exponential Functions
MCR3U: Functions	Many scenarios can be modelled using exponential functions, including:
Applications of Exponential Functions Part 1: Simple Exponential Growth/Decay J. Garvin	 population increase/decrease, radioactive decay, appreciation/depreciation of monetary values, etc.
	In most cases, these functions will have the form $f(t) = a \cdot b^{t/p}$, where:
	 f(t) is the final value, a is the initial value, b is the base (e.g. doubling, halving),
	 <i>t</i> is the time elapsed, and <i>p</i> is the period (the time taken for the action specified
	by the base to occur)
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EXPONENTIAL FUNCTIONS Applications of Exponential Functions Example Alfred has an ant colony, whose population doubles every 5 months. If he begins with 100 ants, approximately how many	Slide 2/11 EXPONENTIAL FUN Applications of Exponential Functions
Applications of Exponential Functions Example Alfred has an ant colony, whose population doubles every 5 months. If he begins with 100 ants, approximately how many ants should there be after 1 year? Assuming no ants are killed or eaten (by ants or by Alfred), we can use the formula $P(t) = 100 \cdot 2^{t/5}$, where P is the	Side 2/11 EXPONENTIAL FUX Applications of Exponential Functions Example How long will it take until the same colony reaches a population of 3200 ants? We want to determine the time, given a final population. Substitute $P(t) = 3200$ into the equation and solve for t. $3200 = 100 \cdot 2^{t/5}$
	Silie 2/11 EXPONENTIAL PUT Applications of Exponential Functions Example How long will it take until the same colony reaches a population of 3 200 ants? We want to determine the time, given a final population. Substitute $P(t) = 3200$ into the equation and solve for t.

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Applications of Exponential Functions

Example How long will it take until the same colony reaches a population of 10 000 ants?

Like before, substitute $P(t) = 10\,000$ into the equation.

 $10\,000 = 100 \cdot 2^{t/5}$ $100 = 2^{t/5}$

Since 100 is not a power of 2, we can't use a change of base like the previous question. Instead, use "guess and check".

Since $2^6 = 64$ and $2^7 = 128$, t should be somewhere between 30 and 35.

When t=33.22, then $P(33.22)\approx 100$. Therefore, it will take slightly longer than 33 months, or $2\frac{3}{4}$ years. . Gavin- Applications of Exponential Functions Slide 5/11

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Example

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to reach 3 200 ants.

Radioactive substances lose mass as they emit particles into the air. The *half-life* of a substance is the time it takes for that substance to decrease by half of its original mass. If 30 grams of a substance has a half-life of 15 years, how much will remain after 50 years?

It will take 25 months, or just over 2 years for the population

Use the equation $M(t) = 30 \left(\frac{1}{2}\right)^{t/15}$, where M is the mass and t is the time in years.

After 50 years, the mass will be $M(50) = 30 \left(\frac{1}{2}\right)^{50/15} \approx 2.98$ grams.

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Example

How much of the radioactive substance from the previous example was present 10 years ago?

Since we want to know a mass in the past, we can use a negative exponent to move "backward" 10 years in time.

Therefore, 10 years ago the mass would have been $M(t)=30\left(rac{1}{2}
ight)^{-10/15}pprox 47.62$ grams.

It is also possible to reverse the process, and treat the scenario as if the substance is doubling rather than halving.

In this case, using the function $M(t) = 30 \cdot 2^{t/15}$ where t = 10, we obtain $M(10) = 30 \cdot 2^{10/15} \approx 47.62$ grams.

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A $\it third$ method is to treat the 30 grams as a final value, and solve for the initial value.

$$30 = a \left(\frac{1}{2}\right)^{10/15}$$
$$a = \frac{30}{\left(\frac{1}{2}\right)^{2/3}}$$
$$a \approx 47.62$$

All three methods are equivalent. Use whichever one is most comfortable.

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Applications of Exponential Functions

Example

A scientist discovers a new strain of bacteria. She isolates 50 bacteria in a petri dish. After 24 hours, the population has grown to 800 bacteria. What is the doubling period?

In this case we want to know the value of p, not t.

$$800 = 50 \cdot 2^{24/p}$$

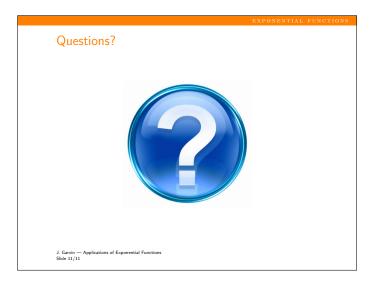
$$16 = 2^{24/p}$$

$$2^{4} = 2^{24/p}$$

$$4 = \frac{24}{p}$$

$$p = 6$$

The bacterial population doubles every 6 hours. J. Garvin — Applications of Exponential Functions Slide 9/11



Applications of Exponential Functions

xample

The value of a painting increases exponentially with time. In 1974, it sold for \$3 000. In 2014 it sold for \$165,000. By how much does the painting increase its value each year?

We know the initial and final values of the painting (\$3000 and \$165000), and also the time elapsed (40 years). We need to determine the base, *b*.

 $165\,000 = 3\,000 \cdot b^{40}$

$$55 = b^{40}$$

 $b = \sqrt[40]{55}$

 $b = \sqrt{35}$ $b \approx 1.1054$

The painting has a value of approximately 110.54% at the end of each year, or an increase of approximately 10.54%. Side 10^{11} Side 10^{11}