MCR3U: Functions

Ambiguous Case of the Sine Law

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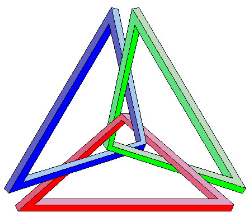
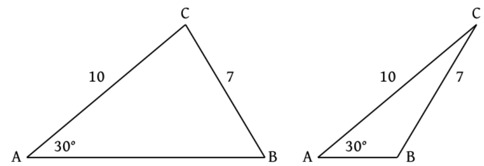


Ambiguous Case of the Sine Law

Consider ΔABC with the following characteristics:

- $\angle A = 30°$
- $|a| = 7$ units
- $|b| = 10$ units

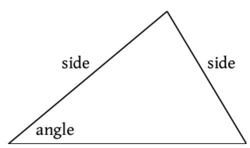
What would this triangle look like?



Conditions For the Ambiguous Case

Using the given information, it was possible to construct two different triangles – one where $\angle B$ was acute, and one where $\angle B$ was obtuse.

This is known as the *ambiguous case*, and occurs when the information is presented in an Angle-Side-Side manner as we move around the triangle.

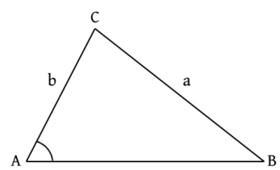


But just because the information is Angle-Side-Side, it does not mean it is ambiguous.

Conditions For the Ambiguous Case

Assume that in ΔABC, we are given $\angle A$, $|a|$ and $|b|$.

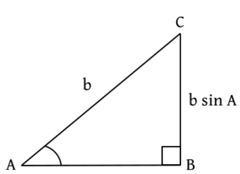
If $|a| > |b|$, then there is only one way to construct the triangle, so it is not ambiguous.



Conditions For the Ambiguous Case

If $|a| < |b|$, we can try to construct a right triangle by constructing the altitude from C as side a.

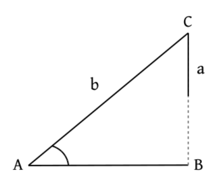
Since $\sin A = \frac{\text{opp}}{\text{hyp}}$ in a right triangle, the length of the altitude is $|b| \sin A$.



There are three possibilities to consider.

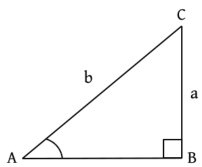
Conditions For the Ambiguous Case

If $|a| < |b| \sin A$, then a does not extend far enough to form a triangle, resulting in an impossible situation.



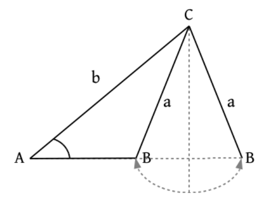
Conditions For the Ambiguous Case

If $|a| = |b| \sin A$, then a is the altitide, and there is one unique right triangle formed.



Conditions For the Ambiguous Case

If $|a| > |b| \sin A$, then there are two possible triangles.



The Ambiguous Case

Given ΔABC where $\angle A$, $|a|$ and $|b|$ are known, there are two possible triangles that can be constructed if $|b| > |a| > |b| \sin A$.

Ambiguous Case of the Sine Law

Example

Determine the $|c|$ in ΔABC if $\angle A = 30°$, $|b| = 10$ and $|a| = 7$.

Check if there are two possible triangles.

$|b| \sin A = 10 \sin 30° = 5$.

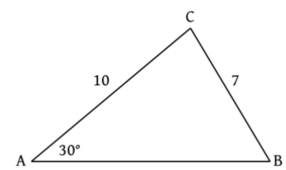
Since $10 > 7 > 5$, two unique triangles can be constructed.

Both cases must be considered as potential solutions to the question.

To determine $|c|$, we must first determine the measure of $\angle C$. This can be done by finding the measure of $\angle B$ using the Sine Law.

Ambiguous Case of the Sine Law

Case 1: $\angle B$ is acute.



$$\frac{\sin 30°}{7} = \frac{\sin B}{10}$$
$$\angle B = \sin^{-1}\left(\frac{10 \sin 30°}{7}\right)$$
$$\approx 45.6°$$

Ambiguous Case of the Sine Law

When $\sin\theta$ is positive, calculators will return the acute angle (in quadrant 1) for $\sin^{-1}(\theta)$.

This is why the Sine Law returned 45.6° in the previous step.

Since $\angle B \approx 45.6°$, $\angle C \approx 180° - 30° - 45.6° \approx 104.4°$.

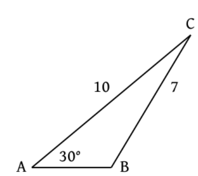
Use the Sine Law to calculate $|c|$.

$$\frac{\sin 30°}{7} \approx \frac{\sin 104.4.6°}{c}$$
$$c \approx \frac{7 \sin 104.4°}{\sin 30°}$$
$$\approx 13.56$$

A possible solution is $|c| = 13.56$ units.

Ambiguous Case of the Sine Law

Case 2: $\angle B$ is obtuse.



In Case 1, $\angle B \approx 45.6°$.

Using this as a reference angle in quadrant 2, an obtuse angle with the same sine ratio as 45.6° is $180° - 45.6° \approx 134.4°$.

Therefore, $\angle C \approx 180° - 30° - 134.4° \approx 15.6°$.

Ambiguous Case of the Sine Law

Use the Sine Law to calculate $|c|$.

$$\frac{\sin 30^\circ}{7} \approx \frac{\sin 15.6^\circ}{c}$$
$$c \approx \frac{7 \sin 15.6^\circ}{\sin 30^\circ}$$
$$\approx 3.76$$

Therefore, $|c|$ is approximately 13.56 or 3.76 units.

Ambiguous Case of the Sine Law

Example

Show that one unique triangle, ΔABC, is formed if $\angle A = 60^\circ$, $|b| = 8$ and $|a| = 4\sqrt{3}$.

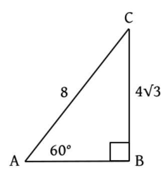
Check if there are two possible triangles.

$|b| \sin A = 8 \sin 60^\circ = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$.

Since $|a| = |b| \sin A$, the side opposite the given angle is the altitude of the triangle.

Therefore, one unique right triangle is formed.

Ambiguous Case of the Sine Law



Questions?