

MCR3U: Functions

## Adding and Subtracting Rational Expressions (Polynomial Denominators)

J. Garvin



Slide 1/11

## Adding and Subtracting Rational Expressions

Recap

Simplify  $\frac{3x+1}{8x} - \frac{2x-5}{6x}$ .

$$\begin{aligned}\frac{3x+1}{8x} - \frac{2x-5}{6x} &= \frac{3(3x+1) - 4(2x-5)}{24x} \\ &= \frac{9x+3-8x+20}{24x} \\ &= \frac{x+23}{24x}, x \neq 0\end{aligned}$$

J. Garvin — Adding and Subtracting Rational Expressions  
Slide 2/11

## Adding and Subtracting Rational Expressions

When rational expressions contain polynomials in their denominators, finding a common denominator can be more challenging.

If necessary, factor each polynomial denominator first, then determine which factors are needed to find the lowest common denominator.

Remember to state restrictions on any factors that may be cancelled from any denominators.

J. Garvin — Adding and Subtracting Rational Expressions  
Slide 3/11

## Adding Rational Expressions

Example

Simplify  $\frac{2}{x+1} + \frac{3}{x-4}$ .

$$\begin{aligned}\frac{2}{x+1} + \frac{3}{x-4} &= \frac{2(x-4) + 3(x+1)}{(x+1)(x-4)} \\ &= \frac{2x-8+3x+3}{(x+1)(x-4)} \\ &= \frac{5x-5}{(x+1)(x-4)}, x \neq -1, x \neq 4\end{aligned}$$

J. Garvin — Adding and Subtracting Rational Expressions  
Slide 4/11

## Adding Rational Expressions

Example

Simplify  $\frac{2x}{x^2-9} + \frac{x+1}{x^2-2x-15}$ .

$$\begin{aligned}\frac{2x}{x^2-9} + \frac{x+1}{x^2-2x-15} &= \frac{2x}{(x+3)(x-3)} + \frac{x+1}{(x+3)(x-5)} \\ &= \frac{2x(x-5) + (x-3)(x+1)}{(x+3)(x-3)(x-5)} \\ &= \frac{2x^2-10x+x^2-2x-3}{(x+3)(x-3)(x-5)} \\ &= \frac{3x^2-12x-3}{(x+3)(x-3)(x-5)}, x \neq \pm 3, x \neq 5\end{aligned}$$

J. Garvin — Adding and Subtracting Rational Expressions  
Slide 5/11

## Subtracting Rational Expressions

Example

Simplify  $\frac{x^2-4}{x^2+4x+4} - \frac{2x^2-5x-12}{x^2-2x-8}$ .

$$\frac{x^2-4}{x^2+4x+4} - \frac{2x^2-5x-12}{x^2-2x-8} = \frac{(x+2)(x-2)}{(x+2)(x+2)} - \frac{(2x+3)(x-4)}{(x+2)(x-4)}$$

We can cancel out the common factors from the numerator and denominator, to find the lowest common denominator.

J. Garvin — Adding and Subtracting Rational Expressions  
Slide 6/11

## Subtracting Rational Expressions

$$\begin{aligned} \frac{(x+2)(x-2)}{(x+2)(x+2)} - \frac{(2x+3)(x-4)}{(x+2)(x-4)} &= \frac{x-2}{x+2} - \frac{2x+3}{x+2} \\ &= \frac{x-2-2x-3}{x+2} \\ &= \frac{-x-5}{x+2}, x \neq -2, x \neq 4 \end{aligned}$$

## Subtracting Rational Expressions

## Example

Simplify  $\frac{x}{x^2+5x+6} - \frac{2}{x^2+3x+2}$ .

$$\begin{aligned} \frac{x}{x^2+5x+6} - \frac{2}{x^2+3x+2} &= \frac{x}{(x+2)(x+3)} - \frac{2}{(x+1)(x+2)} \\ &= \frac{x(x+1) - 2(x+3)}{(x+1)(x+2)(x+3)} \\ &= \frac{x^2+x-2x-6}{(x+1)(x+2)(x+3)} \\ &= \frac{x^2-x-6}{(x+1)(x+2)(x+3)} \end{aligned}$$

## Subtracting Rational Expressions

Factoring the numerator, we can simplify the rational expression even further.

$$\begin{aligned} \frac{x^2-x-6}{(x+1)(x+2)(x+3)} &= \frac{(x-3)(x+2)}{(x+1)(x+2)(x+3)} \\ &= \frac{(x-3)}{(x+1)(x+3)}, x \neq -1, x \neq -2, x \neq -3 \end{aligned}$$

## Adding and Subtracting Rational Expressions

## Your Turn

Simplify  $\frac{x^2+4x-5}{x^2-2x+1} - \frac{2x^2+x-15}{9-x^2}$ .

$$\begin{aligned} \frac{x^2+4x-5}{x^2-2x+1} - \frac{2x^2+x-15}{9-x^2} &= \frac{(x-1)(x+5)}{(x-1)(x-1)} + \frac{(x+3)(2x-5)}{(x+3)(x-3)} \\ &= \frac{(x+5)(x-3) + (x-1)(2x-5)}{(x-1)(x-3)} \\ &= \frac{x^2+2x-15+2x^2-7x+5}{(x-1)(x-3)} \\ &= \frac{3x^2-5x-10}{(x-1)(x-3)}, x \neq 1, x \neq \pm 3 \end{aligned}$$

## Questions?

