ICS3U: Introduction to Computer Science

Decimal, Binary and Hexadecimal Systems

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The Decimal Number System

The number system that we use on a regular basis is the *decimal* system.

It is based on powers of ten.

$$
\begin{array}{cccc}
10^3 & 10^2 & 10^1 & 10^0 \\
1\,000 & 100 & 10 & 1
\end{array}
$$

Any number in the decimal system can be represented as a sum of powers of 10 (multiplied as necessary).

The Decimal Number System

Example

Express $3\,025$ using powers of 10.

$$
\begin{aligned}
3\,025 &= 3000 + 20 + 5 \\
&= 3(1000) + 2(10) + 5(1) \\
&= 3 \cdot 10^3 + 2 \cdot 10^1 + 5 \cdot 10^0
\end{aligned}
$$

The Binary Number System

Unlike us, digital computers do not use the decimal number system.

In fact, computers do not use "numbers" at all. They use electrical signals that are either high (on) or low (off).

It is convenient for us to use numbers to represent these two states, and so we typically use 1 for high and 0 for low.

These two digits form the *binary system*.

The Binary Number System

A single binary digit (0 or 1) is called a *bit* in computer terminology.

A fixed-length string of bits is called a *byte*.

The size of a byte used to be hardware-dependent, but has since been standardized as 8 bits.

Occasionally, an 8-bit byte may be referred to as an *octet*.

A 4-bit string (half a byte) has a name as well: a *nibble*.

Converting Between Decimal and Binary

The binary number system is based on powers of two, similar to how the decimal number system is based on powers of ten.

$$
\begin{array}{cccccccc}
2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\
128 & 64 & 32 & 16 & 8 & 4 & 2 & 1
\end{array}
$$

It uses only the digits 0 and 1 to make all numbers.

As a decimal number made entirely of 9s "rolls over" when 1 is added, a binary number does the same when it is made entirely of 1s.

Thus, the numbers 0-4 in binary are 0, 1, 10, 11 and 100.

We can express decimal values in binary by identfying powers of two.

Converting Between Decimal and Binary

For example, let the binary representation of a number be a string of bits, such as 101.

The rightmost bit represents 2^0, the bit to the left of it represents 2^1, and the leftmost bit represents 2^2.

All bits that are 1 are included, whereas those that are 0 are not.

The decimal equivalent of 101_2 (the subscript 2 indicates that 101 is a binary number) is $2^2 + 2^0 = 4 + 1 = 5_{10}$.

To convert in the other direction, take a number like 6_{10} and identify all powers of two that are included in it.

$6 = 4 + 2 = 2^2 + 2^1$. Therefore, $6_{10} = 110_2$.

We can use one byte (with leading zeroes if necessary) to represent a value between 0 and 255.

Converting Between Decimal and Binary

Example
Express 19_{10} in binary.

$19 = 16 + 2 + 1 = 2^4 + 2^1 + 2^0$. Therefore, $19_{10} = 00010011_2$.

Example
Express 29_{10} in binary.

$29 = 16 + 8 + 4 + 1 = 2^4 + 2^3 + 2^2 + 2^0$. Therefore, $29_{10} = 00011101_2$.

Converting Between Decimal and Binary

Example
Express 00101001_2 in decimal.

The included powers of two are 2^0, 2^3 and 2^5. Therefore, $00101001_2 = 2^0 + 2^3 + 2^5 = 41_{10}$.

Example
Express 10000101_2 in decimal.

The included powers of two are 2^0, 2^2 and 2^7. Therefore, $00101001_2 = 2^0 + 2^2 + 2^7 = 133_{10}$.

The Hexadecimal Number System

Another number system is *hexadecimal*, which uses 16 digits.

Since we do not have single-digit values beyond 9, we use the "numbers" A-F instead.

Dec	Hex	Dec	Hex
0	0	8	8
1	1	9	9
2	2	10	A
3	3	11	B
4	4	12	C
5	5	13	D
6	6	14	E
7	7	15	F

Thus, we count 1, 2, ..., 8, 9, A, B, C, D, E, F, 10, 11, ...

Converting Between Hexadecimal and Binary

The hexadecimal number system is based on powers of sixteen.

16^3	16^2	16^1	16^0
4 096	256	16	1

There is an easy method to convert from hexadecimal to binary.

Converting Between Hexadecimal and Binary

Since $2^4 = 16$, four bits (1 nibble) can be used to represent one hexadecimal digit.

To express a hexadecimal number in binary, split the hexadecimal value into individual digits, then write each digit as a binary nibble.

Example
Express $2A_{16}$ in binary.

The first digit, 2, has a binary value of 0010. The second digit, A, has a binary value of 1010 (decimal 10). Therefore, $2A_{16} = 00101010_2$.

Converting Between Hexadecimal and Binary

To convert from binary to hexadecimal, split the binary number into nibbles. Each nibble is a binary representation of a hexadecimal digit.

Example

Express 01001101_2 in hexadecimal.

The first nibble is 0100, which has a hexadecimal value of 4. The second nibble is 1101 (decimal 13), which has a hexadecimal value of D. Therefore, $01001101_2 = 4D_{16}$.

An alternative way to indicate a hexadecimal value is to use a prefix of 0x. Thus, 0x4D is the same as $4D_{16}$.

Converting Between Hexadecimal and Decimal

We can use binary as an intermediate step for converting from hexadecimal to decimal, or *vice versa*.

Example

Express 17_{16} in decimal.

$17_{16} = 00010111_2$. Converting from binary to decimal, $00010111_2 = 2^4 + 2^2 + 2^1 + 2^0 = 23_{10}$.

Example

Express 26_{10} in hexadecimal.

$26_{10} = 2^4 + 2^3 + 2^1 = 00011010_2$. The first nibble, 0001, is 1 in hexadecimal. The second nibble, 1010 (decimal 10), is A in hexadecimal. Therefore, $26_{10} = 1A_{16}$.

Converting Between Hexadecimal and Decimal

Example

Convert 57_{10} to hexadecimal.

$57_{10} = 2^5 + 2^4 + 2^3 + 2^0 = 00111001_2 = 39_{16}$.

Example

Convert $E5_{16}$ to decimal.

$E5_{16} = 11100101_2 = 2^7 + 2^6 + 2^5 + 2^2 + 2^0 = 229_{10}$.