# ICS3U Case Study: Height Calculator

Let's say that you want to determine the height of a very tall object, such as a flagpole. Perhaps you are able to scale the flagpole like a monkey, but the rest of us might find this task a little too challenging. One way to measure the height is to use a **clinometer**, which is a device that measures the angle of elevation from the horizontal, like in the diagram below.
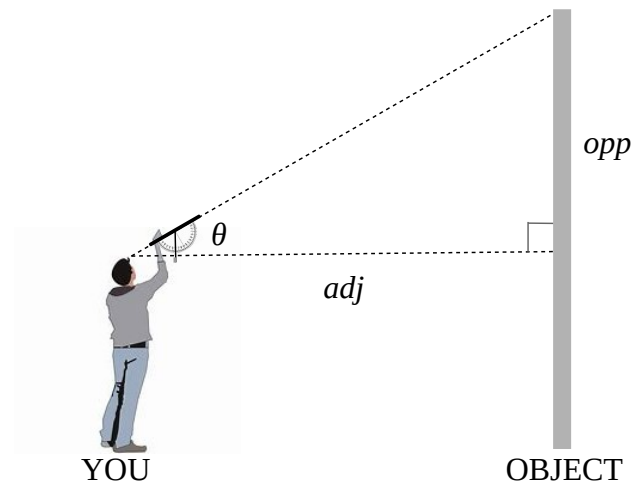
The basic idea is that if we are able to obtain two measures – the horizontal distance from the object, and the angle of elevation – then we can calculate the height of the object using some simple trigonometry. The tangent ratio relates the sides that are opposite and adjacent to the angle of elevation, θ, and so the length of the opposite side of the triangle can be found by the equation

$$\text{opp} = \text{adj} \times \tan \theta .$$

Once the height of the opposite side is known, we can add the height of the clinometer (eye level) to this value to obtain the overall height of the object. For example, consider an observer with an eye level of 175 cm standing 20 m from a tall object. If the observer measures an angle of elevation of $38°$, then the length of the opposite side is

$$\text{opp} = 20 \times \tan 38° \approx 15.63 \text{ m}$$

and the overall height of the object is $15.63 + 1.75 = 17.38$ m. Note that we needed to convert the observer's eye level from centimetres to metres ($175 \div 100 = 1.75$).

A Python program that calculates the height of such an object is below.

```
import math
eye_level = int(input("How many cm from the ground to your eyes? "))
eye_level /= 100
horz_dist = int(input("How many metres from the base of the object? "))
elevation = int(input("Angle of elevation to the top of the object? "))
height = horz_dist * math.tan(math.radians(elevation))
height += eye_level
print("The height of the object is", round(height, 2), "m.")
```

The first line imports the `math` module, since we will be using trigonometric functions that are not loaded up by default. The next two lines read an integer from the user representing the observer's eye level, and convert this value to metres by dividing it by 100 and reassigning the new value to the variable `eye_level`. Note that the program would function in exactly the same way if we asked the user to enter the eye level in metres and used a float to store it, as in the following line.

```
eye_level = float(input("How many m from the ground to your eyes? "))
```

This would eliminate the need to convert the units for the eye level. The next two lines obtain values for the horizontal distance and the angle of elevation. Again, these can be floating point values instead of integers if we wish.

The two lines that follow calculate the height of the opposite side using the tangent ratio, then add the eye level to this value to determine the overall height of the object. Recall that trigonometric ratios use radians in Python instead of degrees, so we need to convert the user's measure using the `radians` function from inside of the `math` module. The last line displays the height, rounded to two decimals.

Running the program, we can verify the calculation that we made earlier.

```
How many cm from the ground to your eyes? 175
How many metres from the base of the object? 20
Angle of elevation to the top of the object? 38
The height of the object is 17.38 m.
```

Exercises

- Write a program that determines the length of a ladder that would be required to reach a desired height against a wall, given an angle of elevation with the ground. The trigonometric ratio that relates the hypotenuse (the ladder) and the opposite side (the wall) is $hyp = opp \div \sin\theta$.

- Write a program that calculates the cost of manufacturing an aquarium. An aquarium is an open-topped rectangular prism. The four vertical faces are made of glass, which costs \$28/m^2. The bottom is made from a sturdier material that costs \$37.50/m^2. Your program should ask the user to input the dimensions (either in metres or in centimetres) and calculate the total cost.

- Write a program that calculates the vertical height, y metres, of a projectile object launched from an initial height, h metres, with an initial velocity, v metres/second, at an angle of θ degrees from the horizontal, at some horizontal position, x metres. The formula for its trajectory is

$$y = h + x\tan\theta - \frac{9.8\,x^2}{2v^2\cos^2\theta}.$$