MPM2D: Principles of Mathematics

Using Trigonometric Ratios

Part 2: Solving For Unknown Angles

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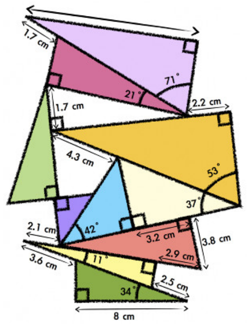
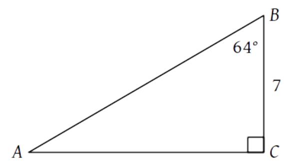


Primary Trigonometric Ratios

Recap

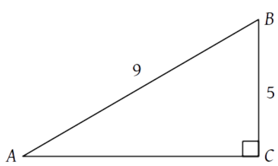
Determine $|AB|$ in the triangle below.



$$\cos B = \frac{\text{adj}}{\text{hyp}}$$
$$\cos 64^\circ = \frac{7}{|AB|}$$
$$|AB| = \frac{7}{\cos 64^\circ}$$
$$|AB| \approx 15.97$$

Primary Trigonometric Ratios

Consider the right triangle below.



How can we find the measure of $\angle A$?

Primary Trigonometric Ratios

We know both $|AB|$ (hypotenuse) and $|BC|$ (opposite), so we can use the sine ratio to relate the sides and angle.

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$
$$\sin A = \tfrac{5}{9}$$

To find the measure of $\angle A$, we use the *inverse of sine* on both sides of the equation.

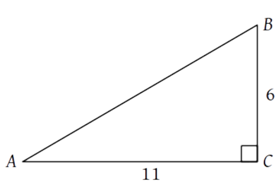
This is similar to how we can square both sides of an equation to eliminate a square root.

$$A = \sin^{-1}\left(\tfrac{5}{9}\right)$$
$$\approx 34^\circ$$

Primary Trigonometric Ratios

Example

Determine the measure of $\angle A$ below.



Since we know the measures of the opposite and adjacent sides, use the inverse of tangent to determine the measure of $\angle A$.

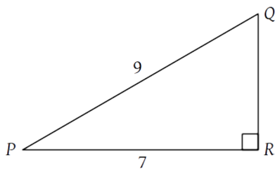
Primary Trigonometric Ratios

$$\tan A = \frac{\text{opp}}{\text{adj}}$$
$$\tan A = \tfrac{6}{11}$$
$$A = \tan^{-1}\left(\tfrac{6}{11}\right)$$
$$A \approx 29^\circ$$

Primary Trigonometric Ratios

Example

Determine the measures of $\angle P$ and $\angle Q$.



The measure of $\angle P$ can be found using the cosine ratio, since we know the lengths of the adjacent side and the hypotenuse are known.

Primary Trigonometric Ratios

$$\begin{aligned} \cos P &= \frac{\text{adj}}{\text{hyp}} \\ \cos P &= \tfrac{7}{9} \\ P &= \cos^{-1}\left(\tfrac{7}{9}\right) \\ P &\approx 39^\circ \end{aligned}$$

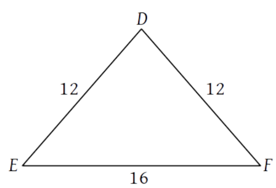
Since the sum of the two non-right angles in a triangle is 90°, $\angle Q \approx 90^\circ - 39^\circ \approx 51^\circ$.

It is possible to use another trigonometric ratio to solve for $\angle Q$, but this would take longer.

Primary Trigonometric Ratios

Example

Determine the measure of $\angle D$.

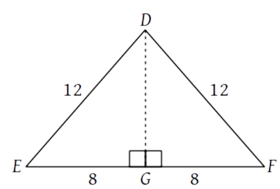


Since ΔDEF does not contain a right angle, we cannot use any trigonometric ratios directly.

Primary Trigonometric Ratios

Since $|DE| = |DF|$, ΔDEF is an isosceles triangle.

This means that the altitude from $\angle D$ is also the median, creating two congruent right triangles as shown.



We can use the sine ratio to calculate the measure of $\angle EDG$.

Primary Trigonometric Ratios

$$\begin{aligned} \sin EDG &= \frac{\text{opp}}{\text{hyp}} \\ \sin EDG &= \tfrac{8}{12} \\ EDG &= \sin^{-1}\left(\tfrac{8}{12}\right) \\ EDG &\approx 42^\circ \end{aligned}$$

Since $\angle EDG = \angle FDG$, $\angle EDF \approx 2 \times 42^\circ \approx 84^\circ$.

Questions?