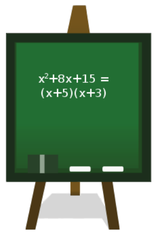
MPM2D: Principles of Mathematics

Distributive Law

Extending the Distributive Law and Special Products

J. Garvin



Products of Two Binomials

Recap

Expand and simplify $(3x-2y)(6x+y)$.

$$\begin{aligned}(3x-2y)(6x+y) &= 18x^2+3xy-12xy-2y^2\\ &= 18x^2-9xy-2y^2\end{aligned}$$

Extending the Distributive Law

The Distributive Law can be extended to multiply any polynomial by another.

As before, each term in the first set of brackets will be multiplied by each term in the second.

For example, $(a+b)(c+d+e)$ will require six multiplications, since there are two terms in the first set of brackets and three in the second, and $2 \times 3 = 6$.

The more terms present, the longer it will take to expand.

Extending the Distributive Law

Example

Expand and simplify $(x-2)(x^2+3x-5)$.

Multiply each term in the first pair of brackets with each term in the second.

$$\begin{aligned}(x-2)(x^2+3x-5) &= x\cdot x^2 + x\cdot 3x - x\cdot 5\\ &\quad -2\cdot x^2 - 2\cdot 3x + 2\cdot 5\\ &= x^3+3x^2-5x-2x^2-6x+10\\ &= x^3+x^2-11x+10\end{aligned}$$

Extending the Distributive Law

Example

Expand and simplify $(x^2+5x-1)(2x^2-4x+3)$.

Multiply each term in the first pair of brackets with each term in the second.

$$\begin{aligned}(x^2+5x-1)(2x^2-4x+3) &= 2x^4-4x^3+3x^2\\ &\quad +10x^3-20x^2+15x\\ &\quad -2x^2+4x-3\\ &= 2x^4+6x^3-19x^2+19x-3\end{aligned}$$

Perfect Squares

Recall that $(ax+c)^2 \neq (ax)^2+c^2$, but can be rewritten to make the Distributive Law clearer.

$$\begin{aligned}(ax+c)^2 &= (ax+c)(ax+c)\\ &= ax\cdot ax + ax\cdot c + c\cdot ax + c\cdot c\\ &= (ax)^2+2acx+c^2\end{aligned}$$

This resulting trinomial is known as a *perfect square*, since it is the result of squaring a binomial.

Note that the coefficient of the middle term is double the product of a and c.

This gives us a quick method for expanding perfect squares.

Perfect Squares

Example

Expand and simplify $(x+5)^2$.

Since this is a perfect square, the coefficient of the middle term will be $2 \times 1 \times 5 = 10$.

The first term will be x^2, and the constant term will be $5^2 = 25$.

Therefore, $(x+5)^2 = x^2 + 10x + 25$.

Check using the Distributive Law.

$$\begin{aligned}(x+5)^2 &= (x+5)(x+5)\\ &= x^2 + 5x + 5x + 25\\ &= x^2 + 10x + 25\end{aligned}$$

Perfect Squares

Example

Expand and simplify $(3x-2)^2$.

Since this is a perfect square, the coefficient of the middle term will be $2 \times 3 \times (-2) = -12$.

The first term will be $(3x)^2 = 9x^2$, and the constant term will be $(-2)^2 = 4$.

Therefore, $(3x-2)^2 = 9x^2 - 12x + 4$.

Check using the Distributive Law.

$$\begin{aligned}(3x-2)^2 &= (3x-2)(3x-2)\\ &= 9x^2 - 6x - 6x + 4\\ &= 9x^2 - 12x + 4\end{aligned}$$

Differences of Squares

In some cases, the product of two binomials results in a binomial with no middle term.

$$\begin{aligned}(ax+c)(ax-c) &= ax \cdot ax - ax \cdot c + ax \cdot c - c \cdot c\\ &= (ax)^2 - c^2\end{aligned}$$

In this case, the two binomials were identical except for the sign between the terms.

The resulting binomial is called a *difference of squares*, since each of the two resulting terms is a square.

It is a common error to refer to differences of squares as perfect squares.

Differences of Squares

Example

Expand and simplify $(x+4)(x-4)$.

Since this will produce a difference of squares, the first term will be x^2 and the constant term will be $-4^2 = -16$.

Therefore, $(x+4)(x-4) = x^2 - 16$.

Check using the Distributive Law.

$$\begin{aligned}(x+4)(x-4) &= x^2 - 4x + 4x - 16\\ &= x^2 - 16\end{aligned}$$

Differences of Squares

Example

Expand and simplify $(5x-3)(5x+3)$.

Since this will produce a difference of squares, the first term will be $(5x)^2 = 25x^2$ and the constant term will be $-3^2 = -9$.

Therefore, $(5x-3)(5x+3) = 25x^2 - 9$.

Check using the Distributive Law.

$$\begin{aligned}(5x-3)(5x+3) &= 25x^2 + 15x - 15x - 9\\ &= 25x^2 - 9\end{aligned}$$

Questions?