MPM2D: Principles of Mathematics

Solving Quadratic Equations

Part 2: Completing the Square

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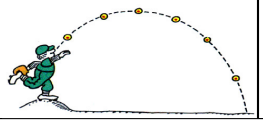


Quadratic Equations

Recap

Solve $x^2 + 14x + 49 = 0$

Factor the perfect square trinomial.

$$(x+7)^2 = 0$$

Isolate x by taking the square root of both sides.

$$x + 7 = 0$$
$$x = -7$$

In this case, -7 is the only solution to the equation.

Quadratic Equations

Example

Solve $(x-8)^2 = 7$.

Isolate x by taking the square root of both sides.

$$x - 8 = \pm\sqrt{7}$$
$$x = 8 \pm \sqrt{7}$$

Therefore, the two solutions are $x = 8 + \sqrt{7}$ and $x = 8 - \sqrt{7}$.

This is a case where a parabola has *irrational* x-intercepts that would not have been found by factoring.

Indeed, the expanded equation $x^2 - 16x + 57 = 0$ is non-factorable.

Quadratic Equations

In the previous example, the expression $(x-8)^2$ is a perfect square.

We can obtain solutions to a non-factorable equation by expressing it as a perfect square, and isolating x.

To do this, we can complete the square, using the same procedure from the last unit.

Note that while it is always possible to express a quadratic using a perfect square (i.e. vertex form), this does not mean that there will be solutions to the equation.

Of course, completing the square can always be done to factorable equations too.

Quadratic Equations

Example

Solve $x^2 + 8x + 8 = 0$.

Complete the square for the simple trinomial.

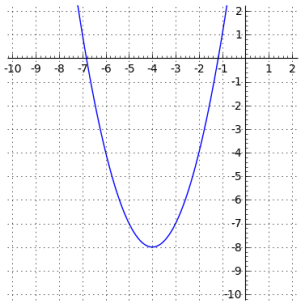
$$x^2 + 8x + 16 - 16 + 8 = 0$$
$$(x+4)^2 - 8 = 0$$

Since the vertex is below the x-axis and opens upward, there will be solutions. Isolate x to find them.

$$(x+4)^2 = 8$$
$$x + 4 = \pm\sqrt{8}$$
$$x = -4 \pm 2\sqrt{2}$$

The solutions are approximately -1.17 and -6.83.

Quadratic Equations



Quadratic Equations

Example

Solve $-2x^2 + 4x + 9 = 0$.

Complete the square for the complex trinomial.

$$\begin{aligned} -2(x^2 - 2x) + 9 &= 0 \\ -2(x^2 - 2x + 1 - 1) + 9 &= 0 \\ -2([x-1]^2 - 1) + 9 &= 0 \\ -2(x-1)^2 + 11 &= 0 \end{aligned}$$

The vertex of the parabola is above the x-axis, and the parabola opens downward, so there will be solutions (x-intercepts) to the equation.

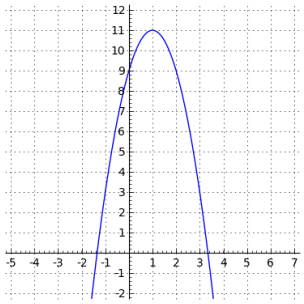
Quadratic Equations

Isolate x.

$$\begin{aligned} -2(x-1)^2 &= -11 \\ x - 1 &= \pm\sqrt{\tfrac{11}{2}} \\ x &= 1 \pm \sqrt{\tfrac{11}{2}} \end{aligned}$$

The solutions are approximately 3.35 and -1.35.

Quadratic Equations



Quadratic Equations

Example

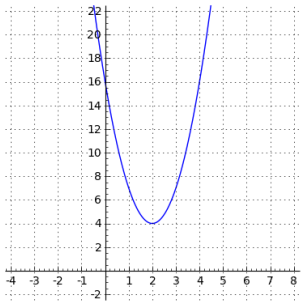
Solve $3x^2 - 12x + 16 = 0$.

Complete the square for the complex trinomial.

$$\begin{aligned} 3(x^2 - 4x) + 16 &= 0 \\ 3(x^2 - 4x + 4 - 4) + 16 &= 0 \\ 3([x-2]^2 - 4) + 16 &= 0 \\ 3(x-2)^2 + 4 &= 0 \end{aligned}$$

The vertex of the parabola is above the x-axis, and the parabola opens upward, so there will be no solutions to the equation.

Quadratic Equations



Questions?