MPM2D: Principles of Mathematics

Shortest Distance

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Altitudes

Recap

Determine the equation of the altitude from $A(-4,7)$ in ΔABC, given $B(-8,0)$ and $C(2,-2)$.

The slope of BC is $m_{BC} = \dfrac{-2-0}{2+8} = -\frac{1}{5}$.

Therefore, the altitude has a slope of 5.

Use the coordinates of A to find its equation.

$$\begin{aligned} 7 &= 5(-4) + b \\ b &= 27 \\ y &= 5x + 27 \end{aligned}$$

Shortest Distance From a Point to a Line

The shortest distance between two points is along a straight line directly between the points.

To find the shortest distance from a point to a line, it is necessary to determine the point on the line that is "most direct" from the given point.

The path to this point will be perpendicular to the line.

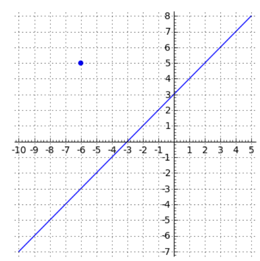
Therefore, the shortest distance is the altitude from the point to the line.

The distance from the given point to the point of intersection of the altitude and the line can then be calculated using the distance formula.

Shortest Distance From a Point to a Line

Example

Determine the shortest distance from $P(-6,5)$ to the line $y = x + 3$.



Shortest Distance From a Point to a Line

The shortest distance from P to the line will be the length of a line segment perpendicular to the line, connecting it to P.

Since the slope of the line is 1, the perpendicular slope is -1.

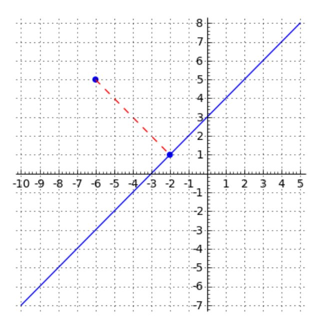
Use the coordinates of P to find the equation of the perpendicular line segment.

$$\begin{aligned} 5 &= -1(-6) + b \\ b &= -1 \\ y &= -x - 1 \end{aligned}$$

Find the point of intersection of the line and line segment.

$$\begin{aligned} x + 3 &= -x - 1 \\ 2x &= -4 \\ x &= -2 \end{aligned} \qquad \begin{aligned} y &= -2 + 3 \\ y &= 1 \end{aligned}$$

Shortest Distance From a Point to a Line



Shortest Distance From a Point to a Line

The line segment intersects the line at $Q(-2, 1)$. Use the distance formula to calculate its length.

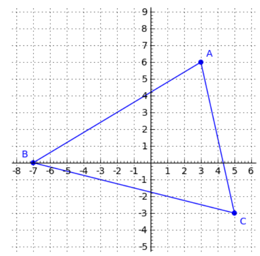
$$|PQ| = \sqrt{(1-(-6))^2 + (-2-5)^2}$$
$$= \sqrt{98}$$
$$= 7\sqrt{2}$$

The shortest distance from $P(-6, 5)$ to the line $y = x + 3$ is $7\sqrt{2} \approx 9.9$ units.

Area of a Triangle

Example

Determine the area of the triangle with vertices $A(3, 6)$, $B(-7, 0)$ and $C(5, -3)$.



Area of a Triangle

The area of a triangle is half the product of its base and height (altitude), or $A = \frac{1}{2}bh$.

A possible altitude for the triangle is from vertex A to BC.

Begin by finding the slope of BC.

$$m_{BC} = \frac{-3-0}{5+7}$$
$$= -\tfrac{1}{4}$$

Now find the equation of the line containing BC.

$$0 = -\tfrac{1}{4}(-7) + b$$
$$b = -\tfrac{7}{4}$$
$$y = -\tfrac{1}{4}x - \tfrac{7}{4}$$

Area of a Triangle

BC has a slope of $-\frac{1}{4}$, so the slope of the altitude is 4.

Determine the equation of the altitude from A.

$$6 = 4(3) + b$$
$$b = -6$$
$$y = 4x - 6$$

Find the intersection of the altitude and BC.

$$-\tfrac{1}{4}x - \tfrac{7}{4} = 4x - 6$$
$$-x - 7 = 16x - 24$$
$$17x = 17$$
$$x = 1$$

$$y = 4(1) - 6$$
$$y = -2$$

The altitude intersects BC at $D(1, -2)$.

Area of a Triangle

The base of the triangle is BC, so calculate its length.

$$|BC| = \sqrt{(-7-5)^2 + (0+3)^2}$$
$$= \sqrt{153}$$

The height of the triangle is the altitude, AD.

$$|AD| = \sqrt{(1-3)^2 + (-2-6)^2}$$
$$= 2\sqrt{17}$$

Use the formula for the area of a triangle.

$$A_{ABC} = \tfrac{1}{2}(2\sqrt{17})(\sqrt{153})$$
$$= 51$$

The triangle has an area of 51 units^2.

Questions?