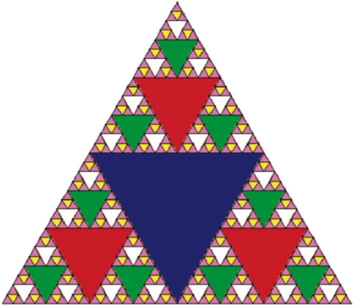
MPM2D: Principles of Mathematics

Ratios and Proportions

J. Garvin



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While this often makes it more convenient to express ratios using a colon for common measurements, it is usually more advantageous for us to use fractions for our calculations.

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Many proportions can be solved by identifying the scale factor, then working out any missing values.

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Since $8 \times \frac{3}{2} = 12$, $k = \frac{3}{2}$.

Therefore, $x = 5 \times k = 5 \times \frac{3}{2} = \frac{15}{2}$.

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Cross-Multiplication

If $\frac{a}{b} = \frac{c}{d}$, then $a \cdot d = b \cdot c$.

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$$\begin{aligned} \frac{x}{12} &= \frac{5}{8} \\ 8x &= 12 \cdot 5 \end{aligned}$$

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$$\frac{x}{12} = \frac{5}{8}$$
$$8x = 12 \cdot 5$$
$$8x = 60$$

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$$\begin{aligned} \frac{x}{4} &= \frac{5}{7} \\ 7x &= 4 \cdot 5 \\ 7x &= 20 \end{aligned}$$

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Proportion problems usually have the format "if a is to b, then c is to. . ."

There may be more than one way to solve the problem. As long as the ratios are valid, solving a proportion using the previous techniques should work.

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$$\frac{s}{12} = \frac{48}{5}$$
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$$5s = 576$$

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$$5s = 576$$
$$s = 115.2$$

A dozen shirts should cost \$115.20.

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$$4\,200b = 16\,800\,000$$
$$b = 4\,000$$

A total of 4 000 bricks were on the pallet.

Questions?