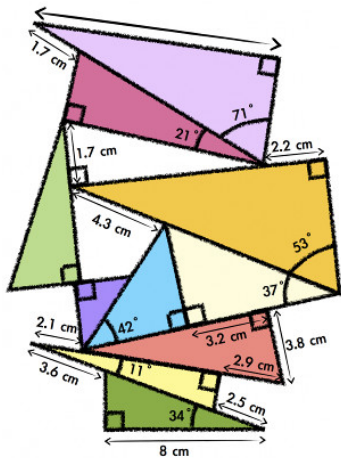


MPM2D: Principles of Mathematics

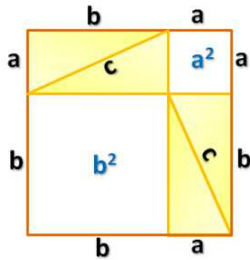
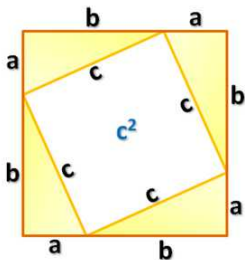
The Pythagorean Theorem

J. Garvin



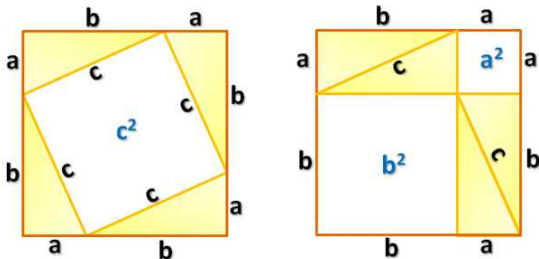
Pythagorean Theorem

Consider four congruent triangles with arms a and b and hypotenuses c . They can be arranged in many ways, including the two below.



Pythagorean Theorem

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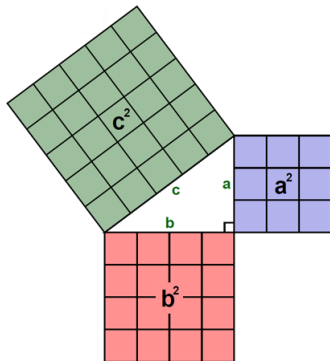


The white area on the left, c^2 , is the same as the sum of the white areas on the right, $a^2 + b^2$.

Pythagorean Theorem

Pythagorean Theorem

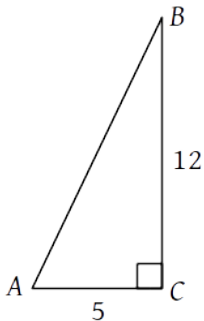
If a and b are the arms in right triangle ABC , and c is the hypotenuse, then $a^2 + b^2 = c^2$.



Pythagorean Theorem

Example

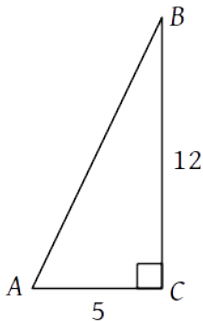
Determine $|AB|$ in the triangle below.



Pythagorean Theorem

Example

Determine $|AB|$ in the triangle below.



$$|AC|^2 + |BC|^2 = |AB|^2$$

$$5^2 + 12^2 = |AB|^2$$

$$169 = |AB|^2$$

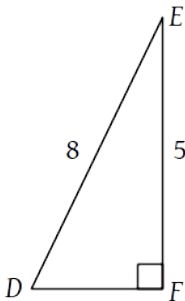
$$\sqrt{169} = |AB|$$

$$13 = |AB|$$

Pythagorean Theorem

Example

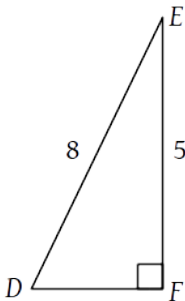
Determine $|DF|$ in the triangle below.



Pythagorean Theorem

Example

Determine $|DF|$ in the triangle below.



$$|DF|^2 + |EF|^2 = |DE|^2$$

$$|DF|^2 + 5^2 = 8^2$$

$$|DF|^2 = 64 - 25$$

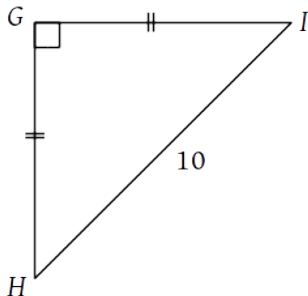
$$|DF| = \sqrt{39}$$

$$|DF| \approx 6.245$$

Pythagorean Theorem

Example

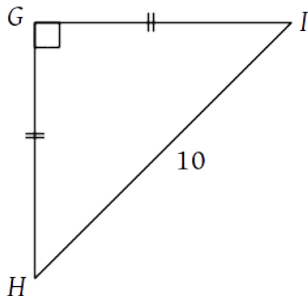
Determine $|GH|$ in the triangle below.



Pythagorean Theorem

Example

Determine $|GH|$ in the triangle below.



$$|GH|^2 + |GI|^2 = |HI|^2$$

$$2|GH|^2 = 10^2$$

$$2|GH|^2 = 100$$

$$|GH|^2 = 50$$

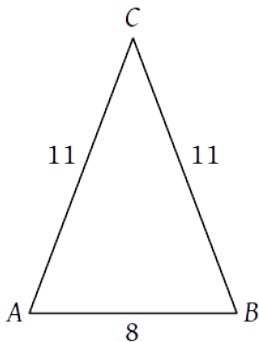
$$|GH| = \sqrt{50}$$

$$|GH| \approx 7.071$$

Pythagorean Theorem

Example

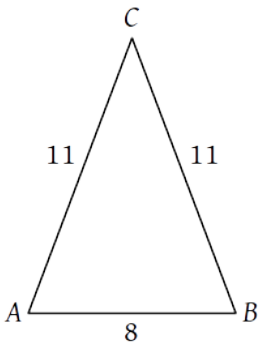
Determine the area of $\triangle ABC$ below.



Pythagorean Theorem

Example

Determine the area of $\triangle ABC$ below.

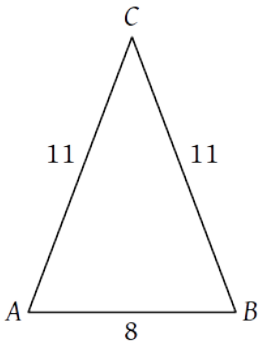


To determine the area of $\triangle ABC$, we first need to determine its height.

Pythagorean Theorem

Example

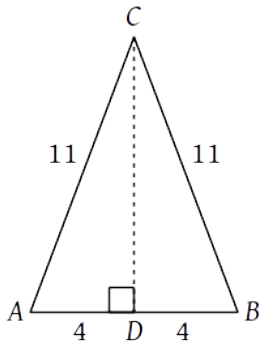
Determine the area of $\triangle ABC$ below.



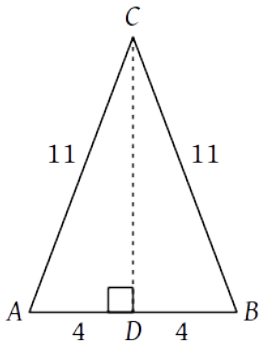
To determine the area of $\triangle ABC$, we first need to determine its height.

Since the triangle is isosceles, the height will *bisect* AB at 90° .

Pythagorean Theorem



Pythagorean Theorem



$$|AD|^2 + |CD|^2 = |AC|^2$$

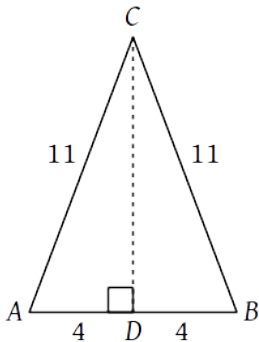
$$4^2 + |CD|^2 = 11^2$$

$$|CD|^2 = 105$$

$$|CD| = \sqrt{105}$$

$$|CD| \approx 10.247$$

Pythagorean Theorem



$$|AD|^2 + |CD|^2 = |AC|^2$$

$$4^2 + |CD|^2 = 11^2$$

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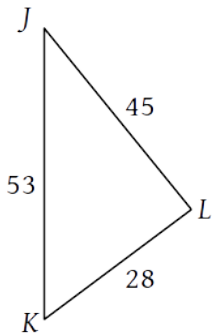
$$|CD| \approx 10.247$$

Using the formula $A = \frac{1}{2}bh$, the area of $\triangle ABC$ is
 $A = \frac{1}{2} \times 8 \times \sqrt{105} \approx 40.988$ square units.

Pythagorean Theorem

Example

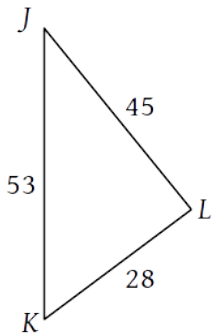
Verify that $\triangle JKL$ contains a right angle.



Pythagorean Theorem

Example

Verify that $\triangle JKL$ contains a right angle.

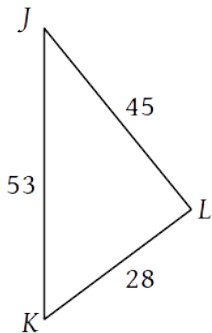


If $\triangle JKL$ contains a right angle, the Pythagorean Theorem will hold true.

Pythagorean Theorem

Example

Verify that $\triangle JKL$ contains a right angle.



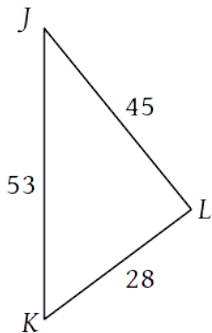
If $\triangle JKL$ contains a right angle, the Pythagorean Theorem will hold true.

The sum of the squares of the arms is $28^2 + 45^2 = 2809$.

Pythagorean Theorem

Example

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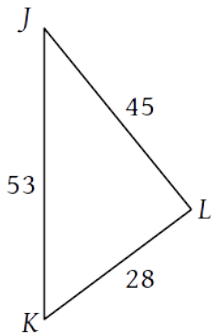
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The square of the hypotenuse is $53^2 = 2809$.

Pythagorean Theorem

Example

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If $\triangle JKL$ contains a right angle, the Pythagorean Theorem will hold true.

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The square of the hypotenuse is $53^2 = 2809$.

Since we obtain the same value, $\angle L = 90^\circ$, as it is across from the hypotenuse.

Questions?

