MPM2D: Principles of Mathematics

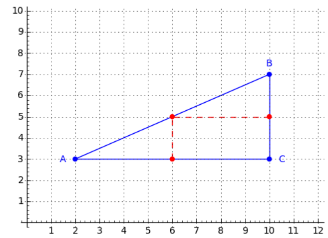
Midpoint of a Line Segment

J. Garvin

Midpoint of a Line Segment

Investigate

Draw right triangle ABC given $A(2,3)$, $B(10,7)$ and $C(10,3)$. Determine the *midpoints* (the points on each line segment that are *equidistant* from the *endpoints*) of AC, BC and AB. How are the coordinates related?



Midpoint of a Line Segment

The midpoint of the horizontal line segment AC is at $(6,3)$.

The x-coordinates of A and C are 2 and 10 respectively, and their average is $\frac{2+10}{2} = 6$.

The y-coordinates of A and C are the same (3), so their average is 3.

Similarly, the midpoint of the vertical line segment BC is at $(10,5)$. The average of the y-coordinates is $\frac{7+3}{2} = 5$, and the average of the x-coordinates is 10.

Midpoint of a Line Segment

This gives us the following two relationships.

Midpoint of a Horizontal Line Segment

If PQ is a horizontal line segment with endpoints at $P(x_1,k)$ and $Q(x_2,k)$, the midpoint M is located at $M\left(\frac{x_1+x_2}{2}, k\right)$.

Midpoint of a Vertical Line Segment

If PQ is a vertical line segment with endpoints at $P(k,y_1)$ and $Q(k,y_2)$, the midpoint M is located at $M\left(k, \frac{y_1+y_2}{2}\right)$.

Midpoint of a Line Segment

Example

Determine the coordinates of the midpoint of $A(3,7)$ and $B(3,19)$.

AB is a vertical line, since the x-coordinates are the same.

The x-coordinate of the midpoint will be 3, and the y-coordinate will be the average of 7 and 19, or $\frac{7+19}{2} = 13$.

Therefore, the midpoint of AB is at $M(3,13)$.

Midpoint of a Line Segment

In the investigation, the midpoint of AB is located at $(6,5)$.

We can verify this by calculating the lengths between the midpoint and each endpoint.

$$\begin{aligned} |AM| &= \sqrt{(6-2)^2+(5-3)^2} \\ &= \sqrt{4^2+2^2} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned} \qquad \begin{aligned} |BM| &= \sqrt{(6-10)^2+(5-7)^2} \\ &= \sqrt{(-4)^2+(-2)^2} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$

Since $|AM| = |BM|$, and M is a point on AB, then M must be the midpoint of AB.

Midpoint of a Line Segment

In the diagram, the midpoint of AB had the same x-coordinate as the midpoint of AC, and the same y-coordinate as the midpoint of BC.

Therefore, we can use the earlier relationships for the x- and y-coordinates of M_{AB}.

Midpoint of a Line Segment

If PQ is a line segment from $P(x_1, y_1)$ to $Q(x_2, y_2)$, then the midpoint of PQ is located at $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

This gives us an algebraic method of finding the midpoint between any two points.

Midpoint of a Line Segment

Example

Determine the coordinates of the midpoint of the line segment connecting $A(4, 9)$ and $B(14, 3)$.

Use the Midpoint Formula with the given coordinates.

$$M_{AB} = \left(\frac{4+14}{2}, \frac{9+3}{2}\right)$$
$$M_{AB} = (9, 6)$$

Midpoint of a Line Segment

Example

Determine the coordinates of the midpoint of the line segment connecting $J(-15, 13)$ and $K(-3, -7)$.

Watch out for negatives.

$$M_{AB} = \left(\frac{-15+(-3)}{2}, \frac{13+(-7)}{2}\right)$$
$$M_{AB} = (-9, 3)$$

Midpoint of a Line Segment

Example

If $M(5, -2)$ is the midpoint of PQ, and P is at $(-8, 11)$, determine the coordinates of Q.

Since we do not know the coordinates of an endpoint, let them be x and y in the Midpoint Formula.

$$(5, -2) = \left(\frac{x+(-8)}{2}, \frac{y+11}{2}\right)$$

This gives the following two linear equations.

$$\begin{aligned} 5 &= \frac{x-8}{2} & \qquad -2 &= \frac{y+11}{2} \\ 10 &= x-8 & -4 &= y+11 \\ x &= 18 & y &= -15 \end{aligned}$$

Q is located at $(18, -15)$.

Midpoint of a Line Segment

Example

Determine if the line $y = -3x + 5$ bisects the line segment AB, given $A(-4, 7)$ and $B(8, -9)$.

If the line bisects AB, it will pass through its midpoint.

The midpoint of AB is at $\left(\frac{-4+8}{2}, \frac{7+(-9)}{2}\right) = (2, -1)$.

Substitute $x = 2$ into the equation of the line to see if it satisfies the equation.

$$\begin{aligned} y &= -3(2) + 5 \\ &= -1 \end{aligned}$$

Since $(2, -1)$ is a valid point on the line, the line bisects AB.

Questions?