MPM2D: Principles of Mathematics

Applications of Linear Systems

Percentages and Mixtures

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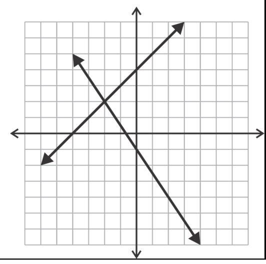

Applications of Linear Systems

Recap

Two numbers have a sum of 61 and a difference of 15. Determine the values of the two numbers.

Let x be one value, and y the other.

The two equations are $x + y = 61$ and $x - y = 15$.

$$\begin{array}{rr} & x + y = 61 \\ - & x - y = 15 \\ \hline & 2y = 46 \\ & y = 23 \end{array} \qquad \begin{array}{rr} & x + y = 61 \\ + & x - y = 15 \\ \hline & 2x \quad = 76 \\ & x \quad = 38 \end{array}$$

The two numbers are 23 and 38.

Percentages and Mixtures

Like the numeric and value-based problems earlier, linear systems can be used to solve problems involving linear relations that deal with percentages and mixtures.

Some of these problems involve an extra step to determine a value before constructing a second equation.

Watch out for percentages that contain decimals. Remember that 8.5% is 0.085 as a decimal, and 0.01% is 0.0001.

Applications Involving Percentages

Example

Two sums of money, totalling \$1600, are invested in separate accounts for a year. One account paid 4%/a interest, while the other paid 6.5%/a. A total of \$94 in interest was earned. How much was invested in each account?

Let x be the sum invested in the account paying 4%/a, and y the sum invested in the account paying 6.5%/a.

From the first sentence, we know that $x + y = 1600$, or $x = 1600 - y$.

From the second, we know that $0.04x + 0.065y = 94$.

The last equation can be multiplied by 200 to give $8x + 13y = 18800$, to avoid working with decimals if desired.

Applications Involving Percentages

Substitute $x = 1600 - y$ for x in $8x + 13y = 18800$.

$$\begin{aligned} 8(1600 - y) + 13y &= 18800 \\ 12800 - 8y + 13y &= 18800 \\ 5y &= 6000 \\ y &= 1200 \end{aligned}$$

Since $x = 1600 - y$, $x = 1600 - 1200 = 400$.

Therefore, \$400 was invested in the 4%/a account, and \$1200 in the 6.5%/a account.

Applications Involving Percentages

Example

Alexander deposits \$500 into one account and \$1200 into another. At the end of one year, his two accounts total \$1754. Gabriella invests \$800 and \$300 into the same two accounts, and earning the same interest after one year. What are the annual interest rates for each account?

Let x be the interest rate for the first account, and y the interest rate for the second.

Since the two deposits total \$1700, the interest earned is $\$1754 - \$1700 = \$54$.

In Alexander's case, $500x + 1200y = 54$.

In Gabriella's case, $800x + 300y = 54$.

Applications Involving Percentages

Multipliy the second equation by 4 to eliminate the y terms.

$$\begin{array}{rrcl} & 500x + 1200y & = & 54 \\ - & 3200x + 1200y & = & 216 \\ \hline & -2700x & = & -162 \\ & x & = & 0.06 \end{array}$$

Multipliy the first equation by 8 and the second by 5 to eliminate the x terms.

$$\begin{array}{rrcl} & 4000x + 9600y & = & 432 \\ - & 4000x + 1500y & = & 270 \\ \hline & 8100y & = & 162 \\ & y & = & 0.02 \end{array}$$

The interest rates are 6%/a and 2%/a.

Mixture Problems

Example

A mixture of nuts is made by combining cashews (\$8.50/kg) with almonds (\$3.25/kg). The resulting mixture has a value of \$4.93/kg. If 50 kg of the mixture is made, what are the masses of the cashews and almonds?

Let c be the mass of the cashews, and a the mass of the almonds.

Since 50 kg are made, $c + a = 50$, or $c = 50 - a$.

50 kg of nut mixture at \$4.93/kg has a value of $50 \times \$4.93 = \$246.50$.

Therefore, $8.50c + 3.25a = 246.50$.

Mixture Problems

Substitute $50 - a$ for c in $8.50c + 3.25a = 246.50$.

$$\begin{aligned} 8.50(50 - a) + 3.25a &= 246.50 \\ 425 - 8.50a + 3.25a &= 246.50 \\ -5.25a &= -178.50 \\ a &= 34 \end{aligned}$$

Since $c = 50 - a$, $c = 50 - 34 = 16$.

Therefore, 16 kg of cashews and 34 kg of almonds are used to create the mixture.

Mixture Problems

Example

A chemist needs 200 mL of a 1.5% acidic solution. Two other acidic solutions – 0.8% and 3.2% – are available for mixing. How much of each solution should be mixed to produce the desired solution?

Let x be the quantity of the 0.8% solution, and y the quantity of the 3.2% solution.

Since the chemist needs 200 mL, $x + y = 200$, or $x = 200 - y$.

1.5% of 200 mL is $0.015 \times 200 = 3$ mL.

Therefore, $0.008x + 0.032y = 3$, or $x + 4y = 375$ without decimals.

Mixture Problems

Substitute $200 - y$ for x in $x + 4y = 375$.

$$\begin{aligned} (200 - y) + 4y &= 375 \\ 3y &= 175 \\ y &= \tfrac{175}{3} \end{aligned}$$

Since $x = 200 - y$, $x = 200 - \frac{175}{3} = \frac{425}{3}$.

Therefore, the chemist needs $\frac{425}{3} \approx 141.7$ mL of the 0.8% solution, and $\frac{125}{3} \approx 58.3$ mL of the 3.2% solution.

Questions?