MPM2D: Principles of Mathematics

# Converting Quadratic Relations to Factored Form

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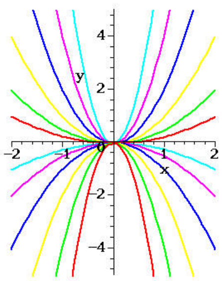







## Quadratic Relations

**Recall**

Graph the relation $y = 2(x+7)(x-1)$.

The $x$-intercepts are at $(-7, 0)$ and $(1, 0)$, and the $y$-intercept is at $(0, -14)$.

The $x$-coordinate of the vertex is $\frac{-7+1}{2} = -3$.

Substitute $x = -3$ into the equation.

$$\begin{aligned} y &= 2(-3+7)(-3-1) \\ &= -32 \end{aligned}$$

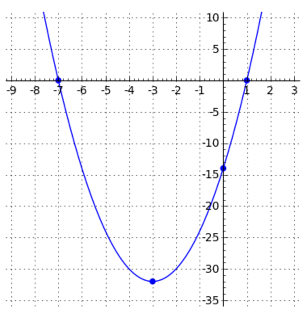
Therefore, the vertex is at $(-3, -32)$.

Plot these four points, adding more as necessary using the step pattern $2, 6, 10, \ldots$.





## Quadratic Relations








## Converting to Factored Form

A quadratic relation may be converted to factored form using any of the factoring techniques covered earlier.

This is an alternative to completing the square, since once we have found the location of the vertex, it is possible to use its coordinates and the vertical stretch factor to rewrite the relation in vertex form.

Both methods have their advantages and disadvantages, but should yield the same results.





## Converting to Factored Form

**Example**

Determine the coordinates of the vertex of the relation $y = x^2 - 8x - 9$.

Since this is a simple trinomial, the quadratic can be represented in factored form as $y = (x+1)(x-9)$.

The $x$-coordinate of the vertex is $\frac{-1+9}{2} = 4$.

The $y$-coordinate of the vertex is $(4+1)(4-9) = -25$.

Therefore, the vertex is at $(4, -25)$, and the relation may also be written as $y = (x-4)^2 - 25$.





## Converting to Factored Form

**Example**

Determine the maximum value of $y = -x^2 - 8x + 33$.

Common factoring $-1$ from all three terms gives $y = -(x^2 + 8x - 33)$.

Factoring the simple trinomial inside of the brackets yields $y = -(x+11)(x-3)$.

The $x$-coordinate of the vertex is $\frac{-11+3}{2} = -4$.

The $y$-coordinate of the vertex is $-(-4+11)(-4-3) = 49$.

Therefore, the maximum value of 49 occurs when $x = -4$.

## Converting to Factored Form

**Example**

Graph the relation $y = 2x^2 - 4x - 30$.

Common factoring 2 from all three terms gives $y = 2(x^2 - 2x - 15)$.

Factoring the simple trinomial inside of the brackets yields $y = 2(x - 5)(x + 3)$.

There $x$-intercepts are at $(5, 0)$ and $(-3, 0)$, and the $y$-intercept is at $(0, -30)$.

The $x$-coordinate of the vertex is $\frac{5-3}{2} = 1$.

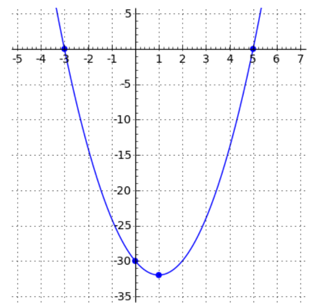
The $y$-coordinate of the vertex is $2(1 - 5)(1 + 3) = -32$.

Therefore, the vertex is at $(1, -32)$.





## Converting to Factored Form








## Converting to Factored Form

**Example**

Graph the relation $y = x^2 - 4x + 5$.

This simple trinomial does not factor, so it cannot be written in the form $y = a(x - r)(x - s)$.

The location of the vertex can still be found by completing the square.

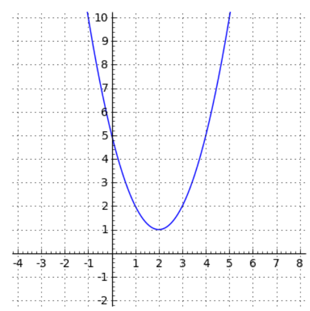
$$\begin{aligned} y &= x^2 - 4x + 4 - 4 + 5 \\ &= (x - 2)^2 + 1 \end{aligned}$$

A quadratic relation can always be written in standard and vertex forms, but it may not be possible to express the relation in factored form.





## Converting to Factored Form








## Questions?