MPM2D: Principles of Mathematics

Completing the Square

Part 2: Complex Trinomials

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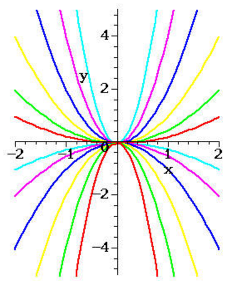


Completing the Square

Recap

Determine the location of the vertex of $y = x^2 + 10x + 19$.

Since $b = 10$, add and subtract a constant of $\left(\frac{10}{2}\right)^2 = 25$ to the trinomial.

$$y = x^2 + 10x + 25 - 25 + 19$$

The first three terms can now be factored as a perfect square, and the last two terms can be combined.

$$y = (x + 5)^2 - 6$$

Therefore, the vertex is at $(-5, -6)$.

Completing the Square

Complex trinomials can be converted to vertex form as well, but since $a \neq 1$, the process involves an additional step.

Since we know how to complete the square using the first two terms of a simple trinomial, we can factor a from the first two terms of a complex trinomial and follow the earlier procedure.

For example, the complex trinomial $5x^2 - 20x + 7$ can be expressed as $5(x^2 - 4x) + 7$.

We can now complete the square on the expression $x^2 - 4x$ inside of the brackets.

Completing the Square

Example

Convert $y = 2x^2 - 20x + 53$ to vertex form.

First, factor 2 out of the first two terms of the trinomial.

$$y = 2(x^2 - 10x) + 53$$

Next, add and subtract a constant of $\left(\frac{-10}{2}\right)^2 = 25$ inside of the brackets.

$$y = 2(x^2 - 10x + 25 - 25) + 53$$

Completing the Square

The first three terms inside of the brackets make a perfect square.

$$y = 2([x - 5]^2 - 25) + 53$$

Before we can combine the constant terms, we must use the distributive property to expand the leading coefficient of 2.

$$\begin{aligned} y &= 2(x - 5)^2 - 50 + 53 \\ &= 2(x - 5)^2 + 3 \end{aligned}$$

Therefore, $y = 2x^2 - 20x + 53$ can be expressed as $y = 2(x - 5)^2 + 3$.

Completing the Square

Example

What is the maximum value of $y = -3x^2 - 18x + 5$?

Since $a = -3$, the parabola opens downward, so a maximum value will occur at its vertex.

Factor -3 from the first two terms of the trinomial.

$$y = -3(x^2 + 6x) + 5$$

Add and subtract a constant of $\left(\frac{6}{2}\right)^2 = 9$ to the trinomial.

$$y = -3(x^2 + 6x + 9 - 9) + 5$$

Completing the Square

The first three terms inside of the brackets make a perfect square.

$$y = -3([x+3]^2 - 9) + 5$$

Use the distributive property to expand the leading coefficient of -3.

$$\begin{aligned} y &= -3(x+3)^2 + 27 + 5 \\ &= -3(x+3)^2 + 32 \end{aligned}$$

Therefore, a maximum value of 32 occurs when $x = -3$.

Completing the Square

Example

Sketch a graph of $y = 2x^2 - 8x$

Since $a = 2$, the parabola opens upward and there will be a minimum at its vertex.

Factor 2 from both terms of the trinomial. Don't be put off that $c = 0$.

$$y = 2(x^2 - 4x)$$

Add and subtract a constant of $\left(\frac{-4}{2}\right)^2 = 4$ to the trinomial.

$$y = 2(x^2 - 4x + 4 - 4)$$

Completing the Square

The first three terms inside of the brackets make a perfect square.

$$y = 2([x-2]^2 - 4)$$

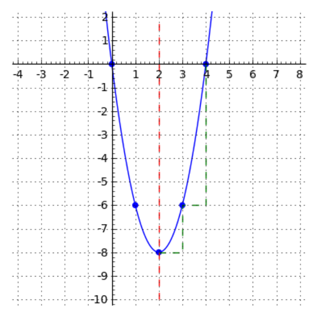
Use the distributive property to expand the leading coefficient of 2.

$$y = 2(x-2)^2 - 8$$

The vertex of the parabola is at $(2, -8)$.

Use the step pattern $2, 6, 10, \ldots$ to graph the parabola.

Completing the Square



Questions?