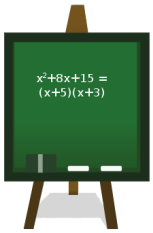
MPM2D: Principles of Mathematics

Factoring Polynomials

Part 1: Common Factoring

J. Garvin



Distributive Law

Recap

Expand and simplify $2x(x^2 - 4x + 5)$.

$$\begin{aligned} 2x(x^2 - 4x + 5) &= 2x \cdot x^2 - 2x \cdot 4x + 2x \cdot 5 \\ &= 2x^3 - 8x^2 + 10x \end{aligned}$$

Common Factoring

Recall that the Greatest Common Factor (GCF) of two or more numbers is the greatest value that divides evenly into those numbers.

For example, the GCF of 6 and 9 is 3, since both 6 and 9 are divisible by 3, but not by a greater number.

Similarly, the GCF of x^3 and x^5 is x^3, since x^3 is divisible by x^3 (obviously), as is x^5: $\frac{x^5}{x^3} = x^2$.

The process of rewriting a polynomial expression as a product of two or more factors is called *factoring*.

If a factor is common to all terms, then we can *common factor* the expression.

Common Factoring

Consider the polynomial expression $6x + 15$.

The GCF of both $6x$ and 15 is 3, since both $6x$ and 15 are evenly divisible by 3, but not by a greater number.

To rewrite the expression in factored form, express it as a product of 3 and some other polynomial. This polynomial is determined by dividing each term by the GCF.

$$\begin{aligned} 6x + 15 &= 3\left(\frac{6x}{3} + \frac{15}{3}\right) \\ &= 3(2x + 5) \end{aligned}$$

Note that the common factor is never discarded. It remains as part of the factored expression.

The Distributive Law can be used to validate the solution.

Common Factoring

Example

Factor $8x - 12$.

The GCF of $8x$ and -12 is 4, so the factored expression will be the product of 4 and another polynomial.

$$\begin{aligned} 8x - 12 &= 4\left(\frac{8x}{4} - \frac{12}{4}\right) \\ &= 4(2x - 3) \end{aligned}$$

Again, note that the common factor, 4, remains as part of the factored expression.

Common Factoring

A common factor must apply to *all* terms in a polynomial expression.

Example

Factor $3x^2 - 15x + 21$.

The GCF of $3x^2$, $-15x$ and 21 is 3, so the factored expression will be the product of 3 and another polynomial.

$$\begin{aligned} 3x^2 - 15x + 21 &= 3\left(\frac{3x^2}{3} - \frac{15x}{3} + \frac{21}{3}\right) \\ &= 3(x^2 - 5x + 7) \end{aligned}$$

Common Factoring

Sometimes it is possible to factor a variable from each term of a polynomial.

Example

Factor $5x^3 - 2x^2 + 3x$.

The GCF of the three terms is x, so the factored expression will be the product of x and another polynomial.

$$\begin{aligned} 5x^3 - 2x^2 + 3x &= x\left(\frac{5x^3}{x} - \frac{2x^2}{x} + \frac{3x}{x}\right) \\ &= x(5x^2 - 2x + 3) \end{aligned}$$

Common Factoring

In other cases, both a variable and a constant make up the GCF.

Example

Factor $15x^4 + 25x^3 - 30x$.

The GCF of the three terms is $5x$. Note that only a single x can be factored, since the exponent on the last term is 1.

$$\begin{aligned} 15x^4 + 25x^3 - 30x &= 5x\left(\frac{15x^4}{5x} + \frac{25x^3}{5x} - \frac{30x}{5x}\right) \\ &= 5x(3x^3 + 5x^2 - 6) \end{aligned}$$

Common Factoring

Expressions involving multiple variables can be handled in the same way.

Example

Factor $16x^3y^2 + 24x^5y^2$.

The GCF of the two terms is $8x^3y^2$. Again, the factor is limited by the lowest exponent on a given variable.

$$\begin{aligned} 16x^3y^2 + 24x^5y^2 &= 8x^3y^2\left(\frac{16x^3y^2}{8x^3y^2} + \frac{24x^5y^2}{8x^3y^2}\right) \\ &= 8x^3y^2(2 + 3x^2) \end{aligned}$$

Note that there are no terms containing y inside of the brackets, since y^2 was completely factored out of all terms.

Common Factoring

Example

Factor $2x^2 + 5x - 3$.

The GCF of the three terms is 1. Therefore, the expression cannot be common factored.

While it is not always possible to find a common factor for the three terms, there are other ways in which polynomials like the one above *can* be factored.

For example, note that the expression can be factored as $(2x - 1)(x + 3) = 2x^2 + 6x - x - 3 = 2x^2 + 5x - 3$.

We will explore some of these other methods over the next few lessons.

Questions?