MPM2D: Principles of Mathematics

Classifying Quadrilaterals

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Classifying Quadrilaterals

Like triangles, we can often classify *quadrilaterals* using slopes, midpoints or lengths.

A quadrilateral is any four-sided polygon.

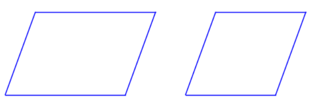
They can be *convex* (no angle is greater than 180°) or concave (at least one angle is greater than 180°).

Special types of quadrilaterals have unique properties.

Classifying Quadrilaterals

A *parallelogram* has two pairs of parallel sides. Opposite sides are equal in length.

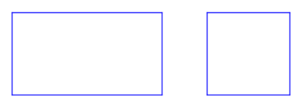
A *rhombus* is a parallelogram in which all four sides are equal in length.



Classifying Quadrilaterals

A *rectangle* is a parallelogram that contains four 90° angles.

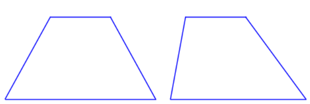
A *square* is a rectangle in which all four sides are equal in length.



Classifying Quadrilaterals

A *trapezoid* has exactly one pair of parallel sides.

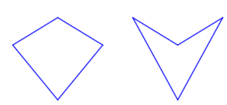
If the two non-parallel sides are equal in length, it is an *isosceles trapezoid*. Otherwise, it is a *scalene trapezoid*.



Classifying Quadrilaterals

A quadrilateral may have two pairs of adjacent sides that have equal lengths.

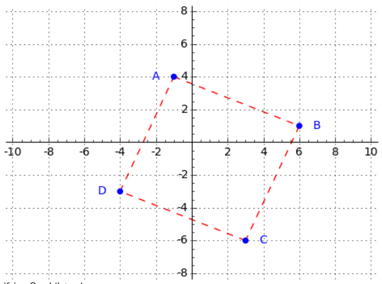
When all interior angles are less than 180°, the quadrilateral is a *kite*. When one angle is greater than 180°, it is a *chevron*.



Classifying Quadrilaterals

Example

Verify that the quadrilateral $ABCD$ with vertices at $A(-1,4)$, $B(6,1)$, $C(3,-6)$ and $D(-4,-3)$ is a square.



Classifying Quadrilaterals

Check if all sides are the same length.

$$|AB| = \sqrt{(6-(-1))^2 + (1-4)^2} = \sqrt{58}$$
$$|BC| = \sqrt{(3-6)^2 + (-6-1)^2} = \sqrt{58}$$
$$|CD| = \sqrt{(-4-3)^2 + (-3-(-6))^2} = \sqrt{58}$$
$$|DA| = \sqrt{(-4-(-1))^2 + (-3-4)^2} = \sqrt{58}$$

Therefore, $ABCD$ is either a square or a rhombus. Next, check if $\angle A$ is a right angle.

$$m_{AB} = \frac{1-4}{6-(-1)} = -\frac{3}{7} \qquad m_{DA} = \frac{-3-4}{-4-(-1)} = \frac{7}{3}$$

Since $AB \perp DA$, $ABCD$ must be a square.

Classifying Quadrilaterals

An alternate solution is to first check for right angles at diagonally opposite vertices.

$$m_{AB} = \frac{1-4}{6-(-1)} = -\frac{3}{7} \qquad m_{DA} = \frac{-3-4}{-4-(-1)} = \frac{7}{3}$$
$$m_{BC} = \frac{-6-1}{3-6)} = \frac{7}{3} \qquad m_{CD} = \frac{-3-(-6)}{-4-3} = -\frac{3}{7}$$

Therefore, $ABCD$ is either a rectangle or a square.

Check the lengths of two adjacent sides.

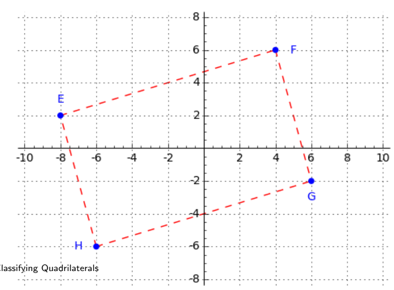
$$|AB| = \sqrt{(6-(-1))^2 + (1-4)^2} = \sqrt{58}$$
$$|BC| = \sqrt{(3-6)^2 + (-6-1)^2} = \sqrt{58}$$

Since $|AB| = |BC|$, $ABCD$ is a square.

Classifying Quadrilaterals

Example

Verify that the quadrilateral $EFGH$ with vertices at $E(-8,2)$, $F(4,6)$, $G(6,-2)$ and $H(-6,-6)$ is a parallelogram, but not a rhombus or a rectangle.



Classifying Quadrilaterals

Check the slopes of the four sides.

$$m_{EF} = \frac{6-2}{4-(-8)} = \frac{1}{3} \qquad m_{FG} = \frac{-2-6}{6-4} = -4$$
$$m_{GH} = \frac{-6-(-2)}{-6-6)} = \frac{1}{3} \qquad m_{HE} = \frac{-6-2}{-6-(-8)} = -4$$

Since $EF \parallel GH$ and $FG \parallel HE$, but $EF \not\perp FG$, $ABCD$ is either a parallelogram or a rhombus.

Check the lengths of two adjacent sides.

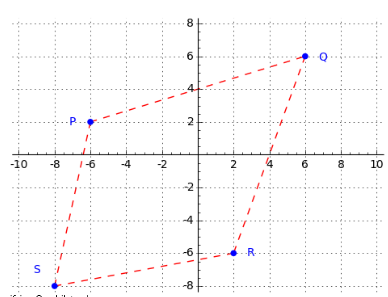
$$|EF| = \sqrt{(4-(-8))^2 + (6-2)^2} = 4\sqrt{10}$$
$$|FG| = \sqrt{(6-4)^2 + (-2-6)^2} = \sqrt{68}$$

Since $|AB| \neq |BC|$, $ABCD$ is a parallelogram.

Classifying Quadrilaterals

Example

Classify the quadrilateral $PQRS$ with vertices at $P(-6,2)$, $Q(6,6)$, $R(2,-6)$ and $S(-8,-8)$.



Classifying Quadrilaterals

Calculate the four side lengths for comparison.

$$|PQ| = \sqrt{(6-(-6))^2 + (6-2)^2} = 4\sqrt{10}$$
$$|QR| = \sqrt{(2-6)^2 + (-6-6)^2} = 4\sqrt{10}$$
$$|RS| = \sqrt{(-8-2)^2 + (-8-(-6))^2} = 2\sqrt{26}$$
$$|SP| = \sqrt{(-6-(-8))^2 + (2-(-8))^2} = 2\sqrt{26}$$

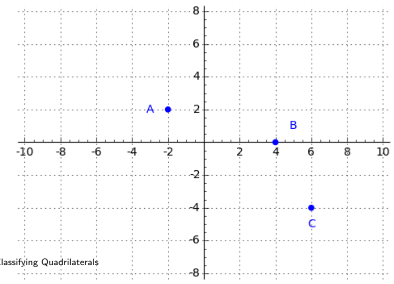
Since there are two pairs of adjacent sides with equal lengths, $PQRS$ is either a kite or a chevron.

Looking at the diagram, it is clear that $PQRS$ is a kite, since there are no angles greater than 180°.

Classifying Quadrilaterals

Example

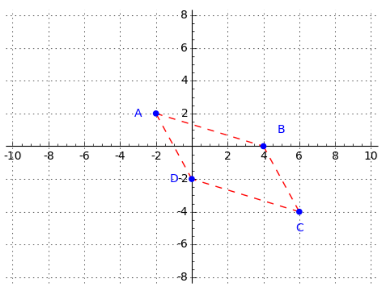
A quadrilateral has three vertices at $A(-2,2)$, $B(4,0)$ and $C(6,-4)$. Determine the coordinates of D so that the quadrilateral is a parallelogram.



Classifying Quadrilaterals

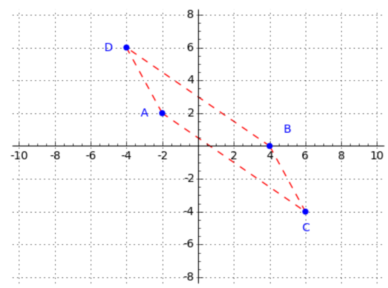
The fourth vertex should be placed so that $AD \parallel BC$.

Since BC has a slope of $\frac{-4-0}{6-4} = -2$, count down from A until $|AD| = |BC|$. This places D at $(0,-2)$.



Classifying Quadrilaterals

Note that this solution is not unique. An alternate location for D can be found by moving upward to $(-4,6)$ instead.



Questions?