MPM2D: Principles of Mathematics

Applications of Sine/Cosine Laws

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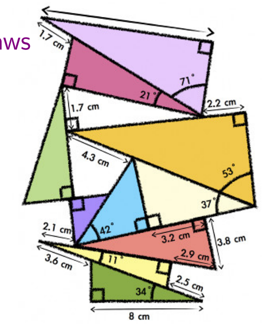


Applications of Sine and Cosine Laws

Many real-world applications involve oblique triangles, where the Sine and Cosine Laws can be used to find certain measurements.

It is important to identify which tool is appropriate. The Cosine Law is used to find a side, given an angle between the other two sides, or to find an angle given all three sides. For all other questions, the Sine Law can be used.

In some cases, it may be necessary to work with multiple triangles by finding a common edge or angle.

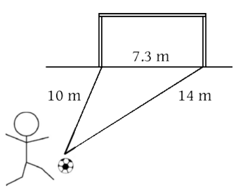
For right triangles, do not forget about simpler tools: Pythagorean Theorem, primary trigonometric ratios, and inverse ratios.

Applications of Sine and Cosine Laws

Example

A soccer player takes a shot on a standard net that is 7.3 m wide. If the player is 10 m from one goalpost and 14 from the other, through what angle can a goal be made?

Sketch a diagram as shown.



Applications of Sine and Cosine Laws

Since three side lengths are known, use the Cosine Law to find the angle.

$$7.3^2 = 10^2 + 14^2 - 2(10)(14)\cos\theta$$
$$\theta = \cos^{-1}\left(\frac{7.3^2 - 10^2 - 14^2}{-2(10)(14)}\right)$$
$$\approx 29.9^\circ$$

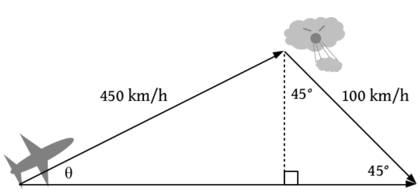
So, the player must shoot the ball through a 30° angle to score a goal.

Applications of Sine and Cosine Laws

Example

A pilot wishes to fly an airplane due East, but a strong wind blowing Southeast at 100 km/h keeps blowing the airplane off-course. If the airplane has a cruising speed of 450 km/h, in what direction should the pilot fly to reach the destination?

A diagram, showing the desired angle θ, is below.



Applications of Sine and Cosine Laws

Use the Sine Law to determine the acute angle θ.

$$\frac{\sin\theta}{100} = \frac{\sin 45^\circ}{450}$$
$$\sin\theta = \frac{100\sin 45^\circ}{450}$$
$$\theta = \sin^{-1}\left(\frac{100\sin 45^\circ}{450}\right)$$
$$\approx 9^\circ$$

The pilot should fly approximately 9° North of East.

Applications of Sine and Cosine Laws

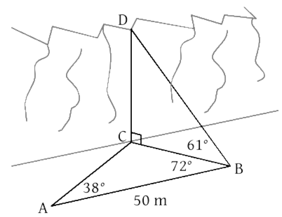
Example

Two surveyors, Alice and Bob, need to determine the height of a steep cliff. They stand 50 m apart where they each have a clear view of the cliff and each other. Bob measures an angle of elevation of 61° from the base of the cliff to its highest point. He also measures the angle between Alice and the base of the cliff as 72°. Alice measures the angle between Bob and the base of the cliff as 38°. How tall is the cliff?

In complex situations like this, it is always important to draw an accurate diagram labelled with all given information.

Applications of Sine and Cosine Laws

In the diagram below, ΔABC lies horizontal on the ground, while ΔBCD projects vertically.



The height of the cliff is $|CD|$, but there is not enough information in the vertical triangle to solve yet.

Applications of Sine and Cosine Laws

Both ΔABC and ΔBCD share a common side, BC.

Determine $\angle ACB$, then use the Sine Law to calculate $|BC|$.

$$\begin{aligned} \angle ACB &= 180^\circ - 38^\circ - 72^\circ \\ &= 70^\circ \\ \frac{|BC|}{\sin 38^\circ} &= \frac{50}{\sin 70^\circ} \\ |BC| &= \frac{50 \sin 38^\circ}{\sin 70^\circ} \\ &\approx 32.76 \text{ m} \end{aligned}$$

Applications of Sine and Cosine Laws

Use the tangent ratio, along with the approximate value of $|BC|$, to determine the height of the cliff, $|CD|$.

$$\begin{aligned} \tan 61^\circ &\approx \frac{|CD|}{32.76} \\ |CD| &\approx 32.76 \tan 61^\circ \\ &\approx 59.1 \text{ m} \end{aligned}$$

So, the height of the cliff is approximately 59.1 m.

Questions?