Applications of Sine and Cosine Laws

Example
A soccer player takes a shot on a standard net that is 7.3 m wide. If the player is 10 m from one goalpost and 14 from the other, through what angle can a goal be made?

Sketch a diagram as shown.

Since three side lengths are known, use the Cosine Law to find the angle.

\[ 7.3^2 = 10^2 + 14^2 - 2(10)(14) \cos \theta \]

\[ \cos \theta = \frac{7.3^2 - 10^2 - 14^2}{-2(10)(14)} \]

\[ \theta \approx 29.9^\circ \]

So, the player must shoot the ball through a 30° angle to score a goal.

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Example
A pilot wishes to fly an airplane due East, but a strong wind blowing Southeast at 100 km/h keeps blowing the airplane off-course. If the airplane has a cruising speed of 450 km/h, in what direction should the pilot fly to reach the destination?

A diagram, showing the desired angle \( \theta \), is below.

Use the Sine Law to determine the acute angle \( \theta \).

\[ \sin \theta = \frac{\sin 45^\circ}{450} \]

\[ \sin \theta = \frac{100 \sin 45^\circ}{450} \]

\[ \theta \approx 9^\circ \]

The pilot should fly approximately 9° North of East.
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Example

Two surveyors, Alice and Bob, need to determine the height of a steep cliff. They stand 50 m apart where they each have a clear view of the cliff and each other. Bob measures an angle of elevation of $61^\circ$ from the base of the cliff to its highest point. He also measures the angle between Alice and the base of the cliff as $72^\circ$. Alice measures the angle between Bob and the base of the cliff as $38^\circ$. How tall is the cliff?

In complex situations like this, it is always important to draw an accurate diagram labelled with all given information.

In the diagram below, $\triangle ABC$ lies horizontal on the ground, while $\triangle BCD$ projects vertically.

The height of the cliff is $|CD|$, but there is not enough information in the vertical triangle to solve yet.

Both $\triangle ABC$ and $\triangle BCD$ share a common side, $BC$. Determine $\angle ACB$, then use the Sine Law to calculate $|BC|$.

$$\angle ACB = 180^\circ - 38^\circ - 72^\circ = 70^\circ$$

$$|BC| = \frac{50}{\sin 38^\circ} \cdot \frac{\sin 70^\circ}{\sin 70^\circ} \approx 32.76 \text{ m}$$

Use the tangent ratio, along with the approximate value of $|BC|$, to determine the height of the cliff, $|CD|$.

$$\tan 61^\circ \approx \frac{|CD|}{32.76}$$

$$|CD| \approx 32.76 \tan 61^\circ \approx 59.1 \text{ m}$$

So, the height of the cliff is approximately 59.1 m.

Questions?