MPM1D: Principles of Mathematics

Working with Exponents

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Rates

Recap

A store sells orange juice in two sizes: 1.8 L for \$2.50 or 3.5 L for \$4.25. Which represents the better bargain?

The unit rate for the 1.8 L bottle is $\frac{2.5}{1.8} \approx 1.39$ \$/L, while it is $\frac{4.25}{3.5} \approx 1.21$ \$/L for the 3.5 L bottle.

Assuming no juice is wasted, the better bargain is the 3.5 L bottle.

Exponents

Recall that an exponent indicates repeated multiplication of a value.

For instance, 5^2 is the same as 5×5, while 3^4 is the same as $3 \times 3 \times 3 \times 3$.

Scientific calculators have buttons for exponentiation, typically labelled something like x^y, y^x, or simply ^.

There may also be shortcuts for common exponents, such as x^2 or x^3.

Since values are being multiplied, exponentiation can result in very large (or small) values.

Exponents

Example

Express $7 \times 7 \times 7$ using an exponent.

Since 7 is multiplied three times, $7 \times 7 \times 7$ can be written with an exponent as 7^3.

Example

Express 4^6 in expanded form.

The exponent indicates that 4 is multiplied 6 times, or $4 \times 4 \times 4 \times 4 \times 4 \times 4$.

Exponents

Example

Simplify, then evaluate, $2 \times 2 \times 2 \times 2 \times 2$.

$2 \times 2 \times 2 \times 2 \times 2 = 2^5$, or 32.

Example

Simplify, then evaluate, $1.8 \times 1.8 \times 1.8 \times 1.8$.

Exponentiation can be done with decimal values in the same way as it is done with integers.

$1.8 \times 1.8 \times 1.8 \times 1.8 = 1.8^4$, or 10.4976.

Fractions and Exponents

What about $\left(\frac{2}{3}\right)^2$?

Recall that $\left(\frac{2}{3}\right)^2$ is the same as $\frac{2}{3} \times \frac{2}{3}$.

Multiplying, we get $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$.

Since $2^2 = 4$ and $3^2 = 9$, the result was that both the numerator and denominator were squared.

In general, we can apply an exponent to each component (numerator or denominator) individually.

Exponents

Example

Evaluate $\left(\frac{2}{5}\right)^3$.

Since $2^3 = 8$ and $5^3 = 125$, $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{8}{125}$.

Example

Evaluate $\left(\frac{1}{10}\right)^6$.

Since $1^6 = 1$ and $10^6 = 10\,000\,000$, $\left(\frac{1}{10}\right)^6 = \frac{1}{10\,000\,000}$.

Negative Exponents

Negative values can also be raised to an exponent.

For example, $(-4)^2$ is the same as $(-4) \times (-4) = 16$, since the product of two negative values is positive.

This is *not* the same as -4^2, which is the same as $-(4 \times 4) = -16$.

In the latter example, the exponent is applied only to the value 4. Be careful.

Negative Exponents

Example

Evaluate $(-5)^3$.

$(-5)^3$ is the same as $(-5) \times (-5) \times (-5) = -125$.

Example

Evaluate -2.5^4.

Since the exponent does not apply to the negative, $-2.5^4 = -39.0625$.

Negative Exponents

We can make some generalizations about the sign of an exponentiated value by examining both the value and the exponent.

Multiplying two negatives produces a positive, multiplying three negatives produces a negative, multiplying four negatives produces a positive, etc.

In general, if a negative value has an even exponent, then its final value will be positive.

If a negative value has an odd exponent, then its final value will be negative.

Negative Exponents

Example

Is the value $(-2)^{17}$ positive or negative?

Since 17 is an odd number, $(-2)^{17}$ will be negative. Its actual value is $-131\,072$.

Example

Is the value -5^8 positive or negative?

Even though the exponent is positive, $-5^8 = -390\,625$, which is negative.

This is because the exponent only applies to the 5 itself, which is then negated. Remember to be careful!

Working with Exponents

Example

Evaluate $2^3 \times 7^2$.

According to the order of operations, exponentiation precedes multiplication.

$$\begin{aligned} 2^3 \times 7^2 &= 8 \times 49 \\ &= 392 \end{aligned}$$

Working with Exponents

Example

Evaluate $3^2 \times 3^3$.

As before, exponentiate first.

$$\begin{aligned} 3^2 \times 3^3 &= 9 \times 27 \\ &= 243 \end{aligned}$$

Note that $243 = 3^5$, and that $3^{2+3} = 3^5$. We will cover this result in more detail in the next lesson.

Questions?