MPM1D: Principles of Mathematics

The Equation of a Line

Part 2: Standard Form

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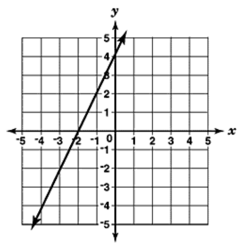


Slope-Intercept Form of a Line

Recap

Graph the line $y = -2x + 4$.

The equation of the line is in slope-intercept form, with a slope of -2 and a y-intercept of 4.

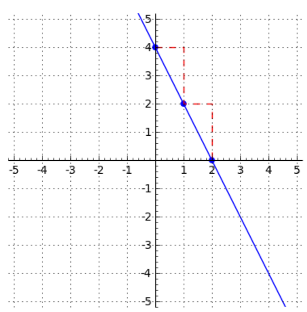
Beginning at $(0, 4)$, count down two units, then right one unit to reach $(1, 2)$.

Repeat this action to move to $(2, 0)$, or additional points as needed.

All points can be connected using a straight line.

Slope-Intercept Form of a Line

A graph is $y = -2x + 4$ is below.



Standard Form of a Line

An alternative to slope-intercept form is *standard form*.

Standard Form of a Line

A line in the form $Ax + By = C$, for some integral values A, B and C, is in *standard form*. By convention, the value of A is positive.

Some sources use a slightly rearranged standard form, $Ax + By + C = 0$.

Note that A, B and C do not correspond directly to the slope or y-intercept of the line, although they are related.

Standard Form of a Line

Example

Convert $3x + 5y = 10$ to slope-intercept form.

Slope-intercept form is $y = mx + b$, so we need to isolate y.

$$\begin{aligned} 3x + 5y &= 10 \\ 5y &= -3x + 10 \\ \frac{5y}{5} &= \frac{-3x}{5} + \frac{10}{5} \\ y &= -\tfrac{3}{5}x + 2 \end{aligned}$$

Thus, $3x + 5y = 10$ describes the same line as $y = -\frac{3}{5}x + 2$.

Standard Form of a Line

Example

Graph the line $2x - 3y = 9$.

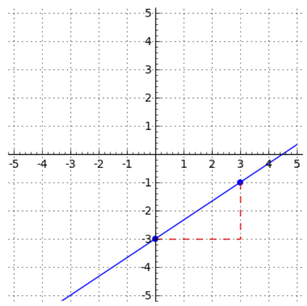
Begin by converting the equation to slope-intercept form.

$$\begin{aligned} 2x - 3y &= 9 \\ -3y &= -2x + 9 \\ \frac{-3y}{-3} &= \frac{-2x}{-3} + \frac{9}{-3} \\ y &= \tfrac{2}{3}x - 3 \end{aligned}$$

The line has a slope of $\frac{2}{3}$ and a y-intercept of -3.

Standard Form of a Line

A graph of $2x - 3y = 9$, or $y = \frac{2}{3}x - 3$, is below.



Standard Form of a Line

Example

Express $y = -\frac{3}{4}x + 2$ in standard form.

Standard form is $Ax + By = C$, so we want to gather the x and y terms on one side of the equation, and eliminate any fractional values.

$$\begin{aligned} y &= -\tfrac{3}{4}x + 2 \\ 4(y) &= 4\left(-\tfrac{3}{4}x + 2\right) \\ 4y &= -3x + 8 \\ 3x + 4y &= 8 \end{aligned}$$

Therefore, $-\frac{3}{4}x + 2$ is $3x + 4y = 8$ in standard form, sometimes expressed as $3x + 4y - 8 = 0$.

Standard Form of a Line

Example

Express $y = \frac{2}{5}x - 3$ in standard form.

Again, gather the x and y terms on one side of the equation, and eliminate any fractional values.

$$\begin{aligned} y &= \tfrac{2}{5}x - 3 \\ 5(y) &= 5\left(\tfrac{2}{5}x - 3\right) \\ 5y &= 2x - 15 \\ -2x + 5y &= -15 \end{aligned}$$

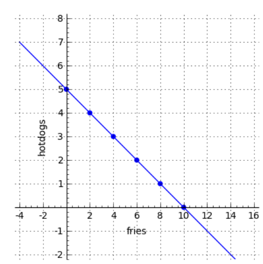
Since the coefficient of x should be positive, multiply both sides by -1.

$$\begin{aligned} -1(-2x + 5y) &= -1(-15) \\ 2x - 5y &= 15 \end{aligned}$$

Standard Form of a Line

Example

A street vendor sells fries for \$2 and hotdogs for \$4. The graph shown below has the equation $2f + 4h = 20$. Interpret this equation, and the graph of the relation itself.



Standard Form of a Line

In the equation, h represents the number of hotdogs purchased and f the number of fries.

The equation, $2f + 4h = 20$, represents all \$20 purchases that can be made. Each point on the line corresponds to a specific purchase.

For example, a customer may buy no fries and five hotdogs, corresponding to the point $(0, 5)$. $2(0) + 4(5) = 20$.

Or, he/she may buy ten fries and no hotdogs, corresponding to $(10, 0)$. $2(10) + 4(0) = 20$.

Or, he/she may buy four fries and three hotdogs, corresponding to $(4, 3)$. $2(4) + 4(3) = 20$.

In all cases, the total money spent is \$20.

Questions?