MPM1D: Principles of Mathematics

Slope of a Line or Line Segment

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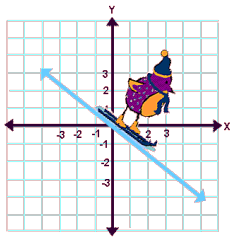
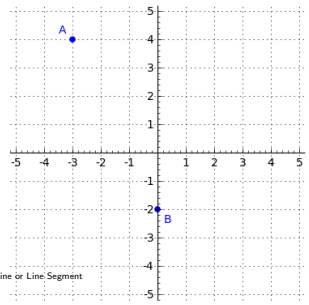


The Cartesian Plane

Recap

Plot the points $A(-3, 4)$ and $B(0, -2)$ on a grid.

A is in Quadrant 2, while B is on the negative y-axis.



Slope of a Line or Line Segment

In the next few units, we will be exploring properties and equations of lines, and occasionally line segments.

While a line continues infinitely in two directions, a line segment has a finite length. Its ends are fixed at two *endpoints*.

The *slope* of a line or line segment is a measure of its steepness: a line with a slope of 5 is steeper than a line with a slope of 4.

Visually, we can see when one line has a steeper slope than another, but how can we represent this mathematically?

Slope of a Line or Line Segment

Fundamentally, the slope of a line can be broken down into two measures: how high it rises vertically, and how far it travels horizontally.

Slope of a Line or Line Segment

The slope, m, of a line or line segment is given by the relationship $m = \frac{\text{rise}}{\text{run}}$, where the rise is the vertical distance from one point to another on the line/segment, and the run is the horizontal distance.

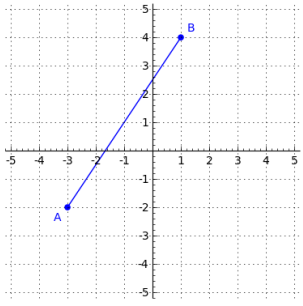
Steep lines will have a large rise and a smaller run, whereas relatively "flat" lines will have a large run and a smaller rise.

When given a line or line segment, determining its slope is usually a matter of identifying the rise and the run using a right triangle.

Slope of a Line or Line Segment

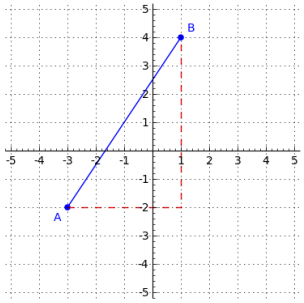
Example

Determine the slope of line segment AB below.



Slope of a Line or Line Segment

Construct a right triangle between A and B on the graph.

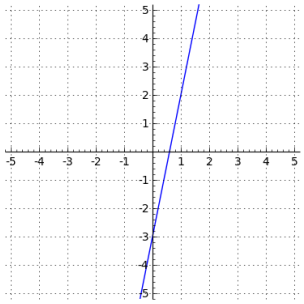


The rise is 6 and the run is 4, so the slope is $m = \frac{6}{4} = \frac{3}{2}$.

Slope of a Line or Line Segment

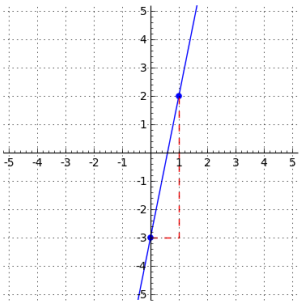
Example

Determine the slope of the line shown.



Slope of a Line or Line Segment

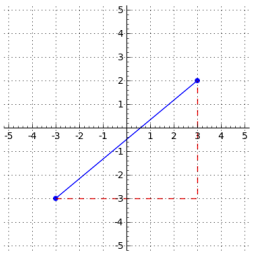
Two known points on the line are $(0, -3)$ and $(1, 2)$.

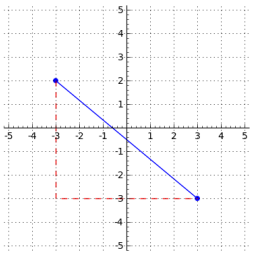


The rise is 5 and the run is 1, so the slope is $m = \frac{5}{1}$, or 5.

Negative Slopes

Consider the two line segments below.





Negative Slopes

Both line segments have a rise of 5 and a run of 6, but have different directions.

Mathematically, we distinguish between the two cases by classifying the slope as either positive or negative.

Positive and Negative Slopes

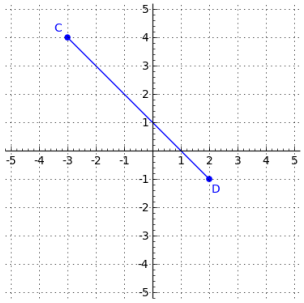
A positive slope indicates that a line/segment rises as it moves from left to right, while a negative slope indicates that it falls as it moves from left to right.

Thus, the line segment on the left had a slope of $\frac{5}{6}$, whereas the one on the right had a slope of $-\frac{5}{6}$.

Negative Slopes

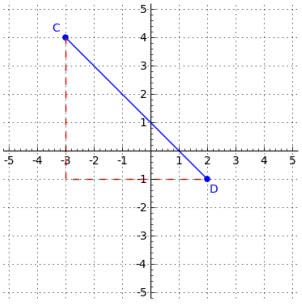
Example

Determine the slope of line segment CD below.



Negative Slopes

Since the line moves downward, the slope is negative.



The rise is 5 and the run is 5, so the slope is $m = -\frac{5}{5} = -1$.

Negative Slopes

Example

Construct a line segment that has one endpoint at $(-4, 2)$, if it has a slope of $-\frac{1}{3}$.

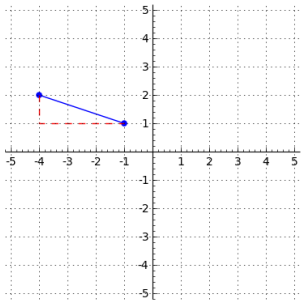
There is not unique solution to this question – many answers are acceptable.

Since the slope is negative, the line must fall as it moves from left to right.

Starting at $(-4, 2)$, move 1 unit down and 3 units right, reaching point $(-1, 1)$.

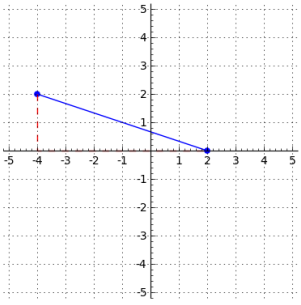
Negative Slopes

The following plot shows a rise of 1 and a run of 3, as required.



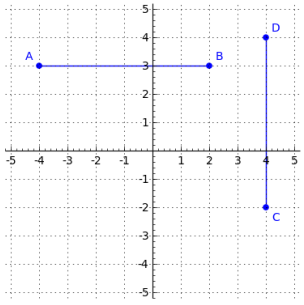
Negative Slopes

The plot below shows an alternate endpoint, $(2, 0)$, with the same slope.



Horizontal and Vertical Lines

Consider the two line segments shown below.



Horizontal and Vertical Lines

Line segment AB has a run of 6, and a rise of 0. Thus, $m = \frac{0}{6} = 0$.

Line segment CD has a run of 0 and a rise of 6. Thus, $m = \frac{6}{0}$, which is mathematically *undefined*.

Slope of a Horizontal or Vertical Line/Segment

If a line or line segment is horizontal, its slope is 0. If a line or line segment is vertical, its slope is undefined.

The endpoints of horizontal line segments will have the same y-coordinate, while the endpoints of vertical line segments will have the same x-coordinate.

Horizontal and Vertical Lines

Example

Determine the slope of the line segment with endpoints $(6, -5)$ and $(6, 13)$.

Since the x-coordinates are the same, the line segment has a run of 0, making it a vertical line. Its slope is undefined.

Example

A horizontal line has passes through $(-8, -3)$. Where does it cross the y-axis?

Since all points on the line segment will have the same y-coordinate, the line will cross the y-axis at $(0, -3)$.

Questions?