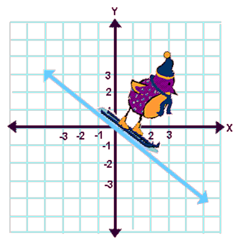
MPM1D: Principles of Mathematics

Partial Variation

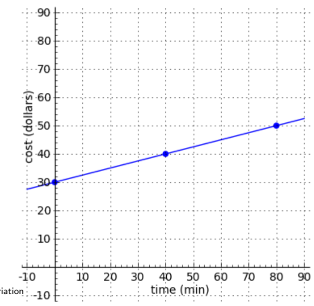
J. Garvin



Slope as a Rate of Change

Recap

The graph below shows the monthly cost of a cell phone. Interpret the slope of the graph as a rate of change.



Partial Variation

The slope of the graph is $\frac{10}{40} = \frac{1}{4}$, so the rate of change is $\frac{1}{4}$ dollars/minute, or 25¢/min.

This means that for every minute used, the cost will increase by 25¢.

Note that in this example, the graph of the relation is a straight line that does not pass through the origin.

This is because there is an initial cost associated with the cell phone plan. In this case, the initial cost is $30, regardless of the number of minutes used.

This type of relationship is known as a *partial variation*.

Partial Variation

Partial Variation

If there is a relationship between two variables where the *dependent variable* is the sum of a constant multiple of the *independent variable* and a constant number, then we refer to the relationship as a partial variation. The graph of such a relation does not pass through the origin.

Partial variations can be written in the form $y = kx + c$, where x and y are variables, and k and c are some real constants.

When $x = 0$, $y = k(0) + c = c$, so the constant number corresponds to the y-intercept of the equation.

We will use this form extensively in this course, and in future courses.

Partial Variation

Example

If the relationship below is a partial variation, complete the table of values and graph the relation.

x	0	1	2	3	4		
y	2	5	8	11	14	23	29

When x changes from 0 to 1, y changes from 2 to 5. Therefore, as x changes by 1, y changes by 3.

The rate of change is $\frac{3}{1} = 3$. Since the relationship is a partial variation, the rate of change is constant.

Thus, when x changes from 2 to 3, y changes from 8 to 11, and when x changes from 3 to 4, y changes from 11 to 14.

Partial Variation

The missing values of x can be found by extending the pattern of increasing y by 3s.

Since $14 + 3 + 3 + 3 = 23$, there are three increases of 3. So the x value must have three increases of 1, or $x = 4 + 1 + 1 + 1 = 7$.

Similarly, $23 + 3 + 3 = 29$, so $x = 7 + 1 + 1 = 9$.

The completed table is below.

x	0	1	2	3	4	7	9
y	2	5	8	11	14	23	29

Partial Variation

Example

A school rents a banquet hall for \$800 for their end-of-year formal. The hall also caters the event, charging the same amount for each plate of food served. A quote for 320 students is \$4 640. How much will it cost if only 250 students attend?

Since there is a fixed cost (the hall rental) and a constant rate of change (the cost per plate), this is an example of partial variation.

Let the relation be $C = kn + h$, where C is the total cost, n is the number of students attending, and h is the cost of the hall.

Thus, an equation for this scenario is $4\,640 = k(320) + 800$.

Partial Variation

We can use the equation to solve for k, the cost per plate.

$$\begin{aligned} 4\,640 &= k(320) + 800 \\ 4\,640 - 800 &= 320k \\ \frac{3\,840}{320} &= \frac{320k}{320} \\ 12 &= k \end{aligned}$$

The hall charges \$12 per plate, so an equation to model the cost is $C = 12n + 800$.

Use this to calculate the cost for 250 students.

$$\begin{aligned} C &= 12(250) + 800 \\ &= 3\,800 \end{aligned}$$

Therefore, it would cost \$3 800 for 250 students.

Partial Variation

Example

For each grammatical error in an essay, a teacher deducts 2 marks. Assume a student begins with a maximum of 30 marks in the grammar category. Create an equation that models a student's grammar score, and use it to calculate his/her score if he/she submits an essay with 7 errors.

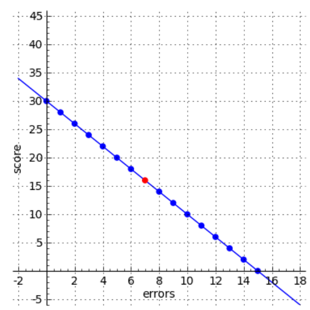
Since the student begins with a fixed value of marks, and marks are deducted at a constant rate, this is an example of partial variation.

The student begins with 30 marks, so the constant number in the equation is 30.

The constant rate of change is 2 marks per error, so an equation is $G = 30 - 2E$, where G is the grammar score and E is the number of errors.

Partial Variation

If the student makes 7 grammatical errors, then his/her score is $G = 30 - 2(7) = 30 - 14 = 16$. A graph confirms this.



Questions?