MPM1D: Principles of Mathematics

Determining Equations of Lines

Part 2: Given Two Points

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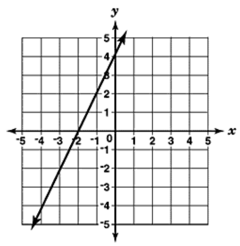

Equation of a Line Given the Slope and a Point

Recap

Determine an equation of a line with a slope perpendicular to the line $y = -6x + 4$, if it passes through the point $(3, 15)$.

The slope of the line with equation $y = -6x + 4$ is -6, so a perpendicular line will have a slope of $\frac{1}{6}$.

Substitute $m = \frac{1}{6}$, $x = 3$ and $y = 15$ into $y = mx + b$.

$$\begin{aligned} 15 &= \tfrac{1}{6}(3) + b \\ 15 &= \tfrac{1}{2} + b \\ 30 &= 1 + 2b \\ 29 &= 2b \\ \tfrac{29}{2} &= b \end{aligned}$$

The equation of the line is $y = \frac{1}{6}x + \frac{29}{2}$.

Equation of a Line Given Two Points

Since a straight line is uniquely defined by any two points, we should be able to determine an equation given two arbitrary points on the Cartesian Plane.

To define a line, we need both its slope and its y-intercept.

We can determine the slope of the line using the slope formula with the coordinates of the two given points.

We can then use either point (both are on the line) along with the slope to determine the equation.

Finding an Equation of a Line Given Two Points

To find an equation of a line passing through two points, (x_1, y_1) and (x_2, y_2), first calculate the slope of the line using $m = \frac{y_2 - y_1}{x_2 - x_1}$. Then, substitute m and any set of given coordinates, (x, y), into $y = mx + b$ to solve for b.

Equation of a Line Given Two Points

Example

Determine an equation of the line that passes through the points $(3, -4)$ and $(9, -2)$.

First, find the slope of the line.

$$\begin{aligned} m &= \frac{-2 - (-4)}{9 - 3} \\ &= \tfrac{1}{3} \end{aligned}$$

Next, substitute $m = \frac{1}{3}$, $x = 3$ and $y = -4$ into $y = mx + b$.

$$\begin{aligned} -4 &= \tfrac{1}{3}(3) + b \\ -4 &= 1 + b \\ -5 &= b \end{aligned}$$

An equation of the line is $y = \frac{1}{3}x - 5$.

Equation of a Line Given Two Points

Note that we obtain the same answer if we use $x = 9$ and $y = -2$, since both points are on the line.

$$\begin{aligned} -2 &= \tfrac{1}{3}(9) + b \\ -2 &= 3 + b \\ -5 &= b \end{aligned}$$

When choosing a point to substitute into $y = mx + b$, choose the one that is easiest to work with. Small values, positive values, or values that "cancel out" fractions are often your best bet.

Equation of a Line Given Two Points

Example

Determine an equation of the line that passes through the points $(12, 5)$ and $(48, 29)$.

First, find the slope of the line.

$$\begin{aligned} m &= \frac{29 - 5}{48 - 12} \\ &= \tfrac{2}{3} \end{aligned}$$

Use $m = \frac{2}{3}$, $x = 12$ and $y = 5$ for smaller values.

$$\begin{aligned} 5 &= \tfrac{2}{3}(12) + b \\ 5 &= 8 + b \\ -3 &= b \end{aligned}$$

An equation of the line is $y = \frac{2}{3}x - 3$.

Equation of a Line Given Two Points

Example

Determine an equation of the line that passes through the points $(-2,-1)$ and $(0,7)$.

Note that the second point has an x-coordinate of zero, indicating that it is the y-intercept. Thus, we just need to find the slope of the line.

$$\begin{aligned} m &= \frac{7-(-1)}{0-(-2)} \\ &= \tfrac{8}{2} \\ &= 4 \end{aligned}$$

Therefore, an equation of the line is $y = 4x + 7$. Remember to keep things simple, and look for shortcuts.

Equation of a Line Given Two Points

Example

Determine the standard form equation of the line that passes through the points $(5,7)$ and $(11,10)$.

Start by determining the slope of the line.

$$\begin{aligned} m &= \frac{10-7}{11-5} \\ &= \tfrac{1}{2} \end{aligned}$$

Next, find the y-intercept of the line.

$$\begin{aligned} 7 &= \tfrac{1}{2}(5) + b \\ 14 &= 5 + 2b \\ 9 &= 2b \\ \tfrac{9}{2} &= b \end{aligned}$$

Equation of a Line Given Two Points

The slope-intercept equation of the line is $y = \frac{1}{2}x + \frac{9}{2}$. Convert this to standard form by eliminating any fractional values and gathering the x and y terms.

$$\begin{aligned} y &= \tfrac{1}{2}x + \tfrac{9}{2} \\ 2y &= x + 9 \\ -x + 2y &= 9 \\ x - 2y &= -9 \end{aligned}$$

The standard form equation of the line is $x - 2y = -9$.

Questions?