MPM1D: Principles of Mathematics

Determining Equations of Lines

Part 1: Given the Slope and a Point

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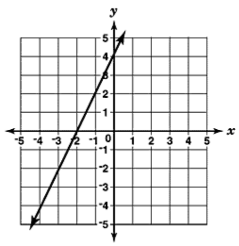

Slope-Intercept Form of a Line

Recap

State an equation of a line perpendicular to $4x + 2y = 5$, if it has the same y-intercept as $8x - 3y = 15$.

Rearrange the first equation into slope-intercept form.

$$\begin{aligned} 4x + 2y &= 5 \\ 2y &= -4x + 5 \\ y &= -2x + \frac{5}{2} \end{aligned}$$

Since the slope of this line is -2, a perpendicular line will have a slope of $\frac{1}{2}$.

Slope-Intercept Form of a Line

The y-intercept of the second equation occurs when $x = 0$.

$$\begin{aligned} 8(0) - 3y &= 15 \\ -3y &= 15 \\ y &= -5 \end{aligned}$$

Thus, the desired equation of the line is $y = \frac{1}{2}x - 5$.

Equation of a Line Given the Slope and a Point

When we are given the slope and the y-intercept of a line, writing its equation in slope-intercept form, $y = mx + b$, is straightforward.

Sometimes, however, we are given the slope and an arbitrary point on the line.

Since a point on the line has coordinates (x, y), we can use these values, along with the slope m, to find an equation of the line.

Finding an Equation of a Line Given Its Slope and a Point

To find the slope-intercept form, $y = mx + b$, of a line with a given slope m and passing through point (x, y), substitute m, x and y into the equation and solve for the y-intercept, b.

Equation of a Line Given the Slope and a Point

Example

Determine an equation of the line with a slope of 3 that passes through the point $(4, 7)$.

Substitute $m = 3$, $x = 4$ and $y = 7$ into $y = mx + b$ and solve for b.

$$\begin{aligned} 7 &= 3(4) + b \\ 7 &= 12 + b \\ 7 - 12 &= b \\ -5 &= b \end{aligned}$$

Therefore, an equation of the line is $y = 3x - 5$.

Equation of a Line Given the Slope and a Point

Example

Determine an equation of the line with a slope of $-\frac{1}{2}$ that passes through the point $(8, 3)$.

Substitute $m = -\frac{1}{2}$, $x = 8$ and $y = 3$ into $y = mx + b$ and solve for b.

$$\begin{aligned} 3 &= -\tfrac{1}{2}(8) + b \\ 3 &= -4 + b \\ 3 + 4 &= b \\ 7 &= b \end{aligned}$$

Therefore, an equation of the line is $y = -\frac{1}{2}x + 7$.

Equation of a Line Given the Slope and a Point

Example

Determine an equation of the line with a slope of 5 that has an x-intercept at -3.

The x-intercept has coordinates $(-3, 0)$, so substitute $m = 5$, $x = -3$ and $y = 0$ into $y = mx + b$.

$$\begin{aligned} 0 &= 5(-3) + b \\ 0 &= -15 + b \\ 15 &= b \end{aligned}$$

Therefore, an equation of the line is $y = 5x + 15$.

Equation of a Line Given the Slope and a Point

Example

Determine an equation of the line that is perpendicular to $3x - 2y = 18$ and has the same x-intercept at $y = -3x + 6$.

Rearrange the first equation to determine the slope.

$$\begin{aligned} 3x - 2y &= 18 \\ -2y &= -3x + 18 \\ y &= \tfrac{3}{2}x - 9 \end{aligned}$$

Since the slope of this line is $\frac{3}{2}$, a perpendicular slope is $-\frac{2}{3}$.

Equation of a Line Given the Slope and a Point

The x-intercept of the second equation occurs when $y = 0$.

$$\begin{aligned} 0 &= -3x + 6 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

Thus, the x-intercept of the line is $(2, 0)$.

Substitute $m = -\frac{2}{3}$, $x = 2$ and $y = 0$ into $y = mx + b$.

$$\begin{aligned} 0 &= -\tfrac{2}{3}(2) + b \\ 0 &= -2(2) + 3b \\ 4 &= 3b \\ \tfrac{4}{3} &= b \end{aligned}$$

Therefore, an equation of the line is $y = -\frac{2}{3}x + \frac{4}{3}$.

Equation of a Line Given the Slope and a Point

Consider a line that passes through two points, (x, y) and (p, q).

Using the slope formula, the slope of this line is $m = \frac{y-q}{x-p}$.

This formula can be rearranged by multiplying both sides by $x - p$.

$$\begin{aligned} m &= \frac{y-q}{x-p} \\ m(x-p) &= y - q \end{aligned}$$

This form is known as *point-slope form* of a line, and is an alternative to slope-intercept form.

Point-Slope Form of a Line

A line with an equation $y - q = m(x - p)$ is in has a slope of m and passes through point (p, q).

Equation of a Line Given the Slope and a Point

Example

Determine an equation of the line with a slope of -2 that passes through the point $(4, 1)$.

Substitute $m = -2$, $p = 4$ and $q = 1$ into $y - q = m(x - p)$ and expand.

$$\begin{aligned} y - 1 &= -2(x - 4) \\ y - 1 &= -2x + 8 \\ y &= -2x + 9 \end{aligned}$$

Therefore, an equation of the line is $y = -2x + 9$.

Equation of a Line Given the Slope and a Point

The same equation could be found by substituting $m = -2$, $x = 4$ and $y = 1$ into $y = mx + b$ instead.

$$\begin{aligned} 1 &= -2(4) + b \\ 1 &= -8 + b \\ 1 + 8 &= b \\ 9 &= b \end{aligned}$$

It is merely a matter of preference which method you use.

Questions?