MPM1D: Principles of Mathematics

Distributive Property

Part 1: Integer Values

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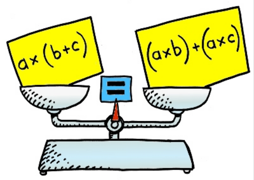

Adding/Subtracting Polynomials

Recap

Simplify $(3x^2+5x)-(7x-9)$.

Remember to reverse the signs when subtracting, and collect like terms.

$$\begin{aligned}(3x^2+5x)-(7x-9)&=3x^2+5x-7x+9\\&=3x^2-2x+9\end{aligned}$$

Distributive Property

Consider the expression $2(3+5)$.

Using the order of operations, $2(3+5)=2(8)=16$.

An alternate way of thinking about this expression is to imagine the 2 being "distributed" to both the 3 and 5 terms of the binomial.

In this case, we get the following.

$$\begin{aligned}2(3+5)&=2(3)+2(5)\\&=6+10\\&=16\end{aligned}$$

The answer is the same, although the process was different.

Distributive Property

In the last example, it would take far more work to evaluate the expression using the second method.

On the other hand, consider the expression $3(x+6)$.

Since x and 6 are not like terms, the binomial cannot be simplified.

It *can*, however, be expressed in an *expanded* form using the alternate method.

$$\begin{aligned}3(x+6)&=3(x)+3(6)\\&=3x+18\end{aligned}$$

Distributive Property

Since this method "distributes" a value to multiple terms of a polynomial, it is called the *distributive property* (sometimes the *distributive rule* or *distributive law*).

In this course, we will be concerned with the distributive property for monomials and polynomials.

Distributive Property for a Monomial and a Polynomial

For a monomial a and polynomial $b+c+d+\ldots$, then $a(b+c+d+\ldots)=ab+ac+ad+\ldots$.

The polynomial can have any number of terms, but we will try to keep things simple in this course.

Distributive Property

Example

Expand $3(x-5)$ using the distributive property.

Multiply both the x and -5 terms of the binomial by 3.

$$\begin{aligned}3(x-5)&=3(x)-3(5)\\&=3x-15\end{aligned}$$

Example

Expand $-2(x^2+4x-3)$ using the distributive property.

Multiply all three terms of the trinomial by -2.

$$\begin{aligned}-2(x^2+4x-3)&=-2(x^2)-2(4x)-2(-3)\\&=-2x^2-8x+6\end{aligned}$$

Distributive Property

Example

Expand $4x(2x^2 - 3x + 5)$ using the distributive property.

This time, we need to use exponent laws.

$$\begin{aligned} 4x(2x^2 - 3x + 5) &= 4x(2x^2) - 4x(3x) + 4x(5) \\ &= 8x^3 - 12x^2 + 20x \end{aligned}$$

Note that the degree of the resulting polynomial expression is the sum of the degrees of the monomial and the polynomial.

Distributive Property

Example

Expand $6x^2(2x^3 + 7x)$ using the distributive property.

Again, use exponent laws to update the variables.

$$\begin{aligned} 6x^2(2x^3 + 7x) &= 6x^2(2x^3) + 6x^2(7x) \\ &= 12x^5 + 42x^3 \end{aligned}$$

The degree of the resulting polynomial expression is 5, which is the sum of the degrees of the monomial (2) and polynomial (3).

Distributive Property

Example

Expand and simplify $5\,(2 + 4(x - 3))$.

This is simply a more complex example involving two applications of the distributive property.

$$\begin{aligned} 5\,(2 + 4(x - 3)) &= 5\,(2 + 4(x) - 4(3)) \\ &= 5\,(2 + 4x - 12) \\ &= 5\,(4x + 2 - 12) \\ &= 5\,(4x - 10) \\ &= 5(4x) - 5(10) \\ &= 20x - 50 \end{aligned}$$

The degree of the resulting polynomial is 5, which is the sum of the degrees of the monomial (2) and polynomial (3).

Distributive Property

Example

Expand and simplify $3x(x - 2) - 2(4x^2 - 7x)$.

Again, apply the distributive property twice. Watch out for the sign change due to subtraction.

$$\begin{aligned} 3x(x - 2) - 2(4x^2 - 7x) &= 3x(x) - 3x(2) - 2(4x^2) + 2(7x) \\ &= 3x^2 - 6x - 8x^2 + 14x \\ &= 3x^2 - 8x^2 - 6x + 14x \\ &= -5x^2 + 8x \end{aligned}$$

Distributive Property

Example

A rectangle has a length that is 10 cm longer than its width. Determine an expression for its perimeter.

We solved a similar problem earlier using the fact that the perimeter of the rectangle is the sum of its side lengths.

If w represents the width of the rectangle, then its length is $w + 10$.

$$\begin{aligned} w + (w + 10) + w + (w + 10) &= (w + w + w + w) + (10 + 10) \\ &= 4w + 20 \end{aligned}$$

Distributive Property

The same problem can be solved using the distributive property by noting that the perimeter is made up of two equal widths, and two equal lengths.

$$\begin{aligned} 2(w) + 2(w + 10) &= 2w + 2w + 2(10) \\ &= 4w + 20 \end{aligned}$$

Both approaches yield the same answer. It is largely a matter of preference as to which method is "better."

Questions?