MPM1D: Principles of Mathematics

Direct Variation

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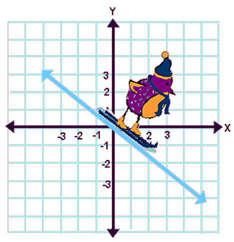
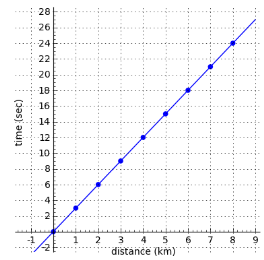


Slope as a Rate of Change

Recap

The graph below shows the time it takes for an observer d km away to hear thunder after witnessing a lightning strike. Interpret the slope of the graph as a rate of change.



Slope as a Rate of Change

The slope of the graph is $\frac{3}{1} = 3$, so the rate of change is 3 sec/km.

This means that for every kilometre away from the lightning strike, it will take an additional 3 seconds for the sound to reach the observer's ears.

Note that in this example, the graph of the relation is a straight line that passes through the origin.

This should make sense, since the speed of sound is constant (sort of), and it would take zero seconds for the sound to travel to an observer at the strike location.

This type of relationship is known as a *direct variation*.

Direct Variation

Direct Variation

If there is a relationship between two variables where one variable is a constant multiple of the other, then we refer to the relationship as a direct variation. The graph of such a relation passes through the origin.

Direct variations can be written in the form $y = kx$, where x and y are variables and k is some real constant.

k is sometimes referred to as the *constant of variation*, but we have already encountered it as a *constant rate of change* or as a *slope* of a line.

Direct Variation

Example

Emma earns \$12 per hour at a department store. Determine an equation that models her pay after t hours, and illustrate the relationship with a graph.

The relationship is a direct variation – Emma's pay depends solely on the number of hours worked.

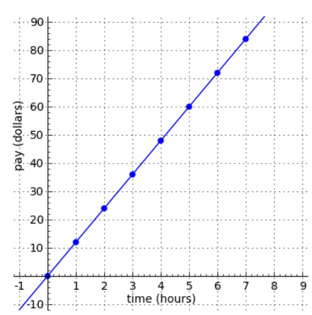
An equation that models her pay is $P = 12t$, where P is her pay in dollars.

If Emma were to work 0 hours, she would earn a total of $P = 12 \times 0 = \$0$.

If she were to work 7 hours, she would earn a total of $P = 12 \times 7 = \$84$.

Direct Variation

A graph of Emma's pay is below. Note that it passes through the origin.



Direct Variation

Example

After 40 minutes, a car travelling at a constant speed is 58 km from its departure point. How long will it take to travel to its destination, 261 km away?

Since the car is 0 km from its departure point when it begins its travel, this is an example of direct variation – the distance depends on how long the car is travelling.

To determine the rate of change, use the relationship for speed, distance and time.

$$s = \frac{d}{t}$$
$$s = \frac{58}{40}$$
$$s = 1.45$$

Direct Variation

Therefore, the speed of the car is 1.45 km/min.

Use this as the constant value in an equation to calculate the time required to travel 261 km.

$$t = \frac{d}{s}$$
$$t = \frac{261}{1.45}$$
$$t = 180$$

Therefore, it will take 180 minutes, or 3 hours, to reach the destination.

Direct Variation

Note that this could have been solved using km/h for a more "meaningful" speed, by representing 40 minutes as $\frac{2}{3}$ of an hour.

$$s = \frac{58}{\frac{2}{3}}$$
$$s = \frac{58}{1} \times \frac{3}{2}$$
$$s = 87$$

The car's speed is 87 km/h. Since $3 \times 87 = 261$, it will take 3 hours to reach its destination.

Questions?