MHF4U: Advanced Functions

Solving Trigonometric Equations Using Identities

Part 1: Tangent and Pythagorean Identities

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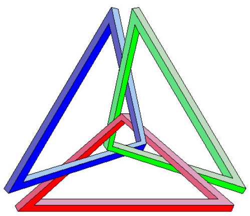


Solving Trigonometric Equations

Example

Solve $2\cos^2 x + \sin x - 1 = 0$ on $[0, 2\pi]$.

Since the equation involves both sine and cosine, use the Pythagorean Identity to express $\cos^2 x$ in terms of $\sin^2 x$ instead.

$$\begin{aligned} 2(1-\sin^2 x) + \sin x - 1 &= 0 \\ 2 - 2\sin^2 x + \sin x - 1 &= 0 \\ 2\sin^2 x - \sin x - 1 &= 0 \end{aligned}$$

Solving Trigonometric Equations

Factor the equation to find the zeroes.

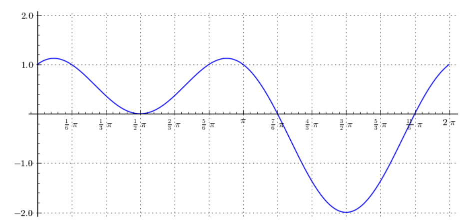
$$\begin{aligned} (\sin x - 1)(2\sin x + 1) &= 0 \\ \sin x &= 1, -\tfrac{1}{2} \end{aligned}$$

When $\sin x = 1$, $x = \frac{\pi}{2}$.

When $\sin x = -\frac{1}{2}$, $x = \frac{7\pi}{6}, \frac{11\pi}{6}$.

Thus, the three solutions are $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

Solving Trigonometric Equations



Solving Trigonometric Equations

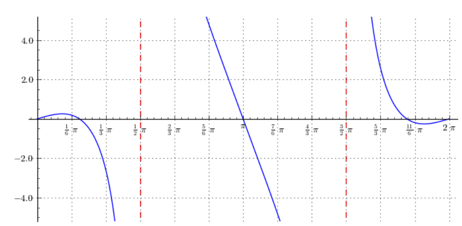
Example

Solve $5\sin x - 4\tan x = 0$ on $[0, 2\pi]$.

$$\begin{aligned} 5\sin x - 4\tan x &= 0 \\ 5\sin x &= 4\tan x \\ \frac{\sin x}{\tan x} &= \frac{4}{5} \\ \cos x &= \frac{4}{5} \\ x &= \cos^{-1}\left(\frac{4}{5}\right) \\ x &\approx 0.6435, 5.6397 \end{aligned}$$

Therefore, the two solutions are $x = 0.6435, 5.6397$.

Solving Trigonometric Equations



Wait, what?

The graph shows that there are 5 solutions, so where did the other three go?

Solving Trigonometric Equations

By replacing $\tan x$ with $\frac{\sin x}{\cos x}$, we inadvertently threw away three solutions.

Here is a better solution that uses the fact that $\sin x = \cos x \tan x$.

$$\begin{aligned} 5\cos x \tan x - 4\tan x &= 0 \\ \tan x(5\cos x - 4) &= 0 \end{aligned}$$

Thus, there are solutions when $\tan x = 0$, so $x = 0, \pi, 2\pi$.

The other two solutions, when $5\cos x - 4 = 0$, are the ones found earlier, $x \approx 0.6435, 5.6397$.

Therefore, the five solutions are $x = 0, \pi, 2\pi$ and $x \approx 0.6435, 5.6397$.

Solving Trigonometric Equations

Example

Solve $3\sin x - \cot x = 0$ on $[0, 2\pi]$.

Use the fact that $\cot x = \frac{\cos x}{\sin x}$, and apply the Pythagorean Identity.

$$\begin{aligned} 3\sin x - \frac{\cos x}{\sin x} &= 0 \\ 3(1 - \cos^2 x) - \cos x &= 0 \\ 3\cos^2 x + \cos x - 3 &= 0 \end{aligned}$$

Solving Trigonometric Equations

Use the quadratic formula to find the zeroes.

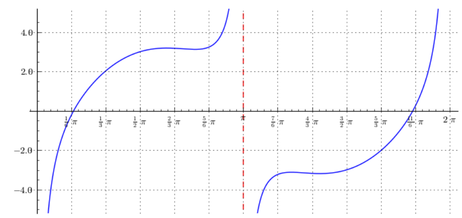
$$\begin{aligned} \cos x &= \frac{-1 \pm \sqrt{1^2 - 4(3)(-3)}}{2(3)} \\ \cos x &= \frac{-1 \pm \sqrt{37}}{6} \end{aligned}$$

When $\cos x = \frac{-1+\sqrt{37}}{6}$, $x \approx 0.56, 5.72$.

$\cos x \neq \frac{-1-\sqrt{37}}{6}$, since $\frac{-1-\sqrt{37}}{6} \approx -1.18$, which is outside of the range of $\cos x$.

Thus, the two solutions are $x \approx 0.56, 5.72$.

Solving Trigonometric Equations



Questions?