MHF4U: Advanced Functions

Solving Rational Inequalities

Part 1: Simple Inequalities

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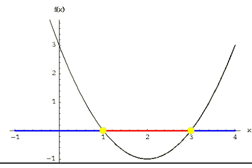


Rational Inequalities

Rational inequalities can be solved using similar techniques for solving polynomial inequalities: cases or intervals.

Recall the rules for solving inequalities.

Rules for Solving Inequalities

- The same value may be added to, or subtracted from, both sides of an inequality.
- Each side of an inequality may be multiplied, or divided, by the same positive value.
- Each side of an inequality may be multiplied, or divided, by the same negative value *if the inequality is reversed*.
- If each side of an inequality has the same sign, the reciprocal of each side may be taken *if the inequality is reversed*.

Solving Rational Inequalities Using Cases

Example

Solve $\frac{3}{x-2} > -4$ using cases.

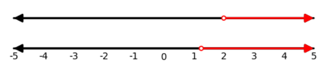
Since $x - 2 \neq 0$, there are two cases to consider.

Case 1: $x - 2 > 0$, or $x > 2$.

$$\begin{aligned} \frac{3}{x-2} &> -4 \\ 3 &> -4(x-2) \\ 3 &> -4x + 8 \\ -5 &> -4x \\ \tfrac{5}{4} &< x \end{aligned}$$

Solving Rational Inequalities Using Cases

Consider the two intervals on a number line.



Since $x > 2$ is common, it is a solution to the inequality.

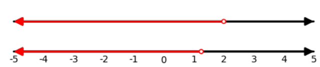
Solving Rational Inequalities Using Cases

Case 2: $x - 2 < 0$, or $x < 2$.

$$\begin{aligned} \frac{3}{x-2} &> -4 \\ 3 &< -4(x-2) \\ 3 &< -4x + 8 \\ -5 &< -4x \\ \tfrac{5}{4} &> x \end{aligned}$$

Solving Rational Inequalities Using Cases

Consider the two intervals on a number line.

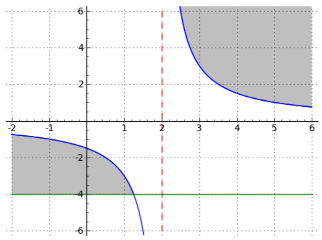


Since $x < \frac{5}{4}$ is common, it is a solution to the inequality.

The solution, then, is $\frac{3}{x-2} > -4$ on $\left(-\infty, \frac{5}{4}\right) \cup (2, \infty)$.

A graph confirms these intervals.

Solving Rational Inequalities Using Cases



While this method works, it can be tedious and difficult to follow at times.

Solving Rational Inequalities Using Intervals

Example

Solve $\frac{x^2 - 6x + 8}{2x^2 + 5x - 3} \leq 0$ using intervals.

Begin by factoring the numerator and denominator to determine any vertical asymptotes or x-intercepts that define intervals.

$$\frac{x^2 - 6x + 8}{2x^2 + 5x - 3} \leq 0$$
$$\frac{(x-4)(x-2)}{(2x-1)(x+3)} \leq 0$$

There are vertical asymptotes at $x = -3$ and $x = \frac{1}{2}$, and x-intercepts at $x = 2$ and $x = 4$.

Solving Rational Inequalities Using Intervals

Set up a table with five intervals.

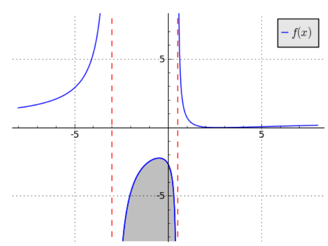
Interval	$(-\infty, -3)$	$(-3, \frac{1}{2})$	$(\frac{1}{2}, 2)$	$(2, 4)$	$(4, \infty)$
x	-4	0	1	3	5
Sign of $P(x)$	$+$	$-$	$+$	$-$	$+$

The rational function is less than zero on two intervals, and equal to zero at the two roots.

Therefore, $\frac{x^2 - 6x + 8}{2x^2 + 5x - 3} \leq 0$ on $\left(-3, \frac{1}{2}\right) \cup [2, 4]$.

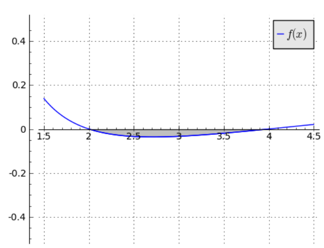
Again, graphing confirms the intervals.

Solving Rational Inequalities Using Cases



It is hard to see the detail between the two x-intercepts at this scale, so zoom in for clarity.

Solving Rational Inequalities Using Cases



Questions?