MHF4U: Advanced Functions

Radian Measure

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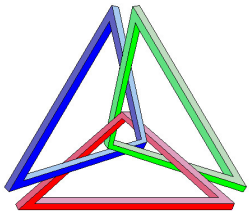


Radian Measure

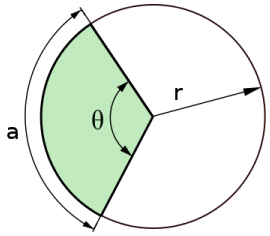
In the past, we have worked exclusively with degrees as our unit of measurement for angles.

An alternative measurement system uses *radians*, rather than degrees.

One radian is defined as the measure of the angle that is subtended by an arc that has a length equal to that of the radius.

Radian Measure

Let r be the radius of a circle, and let a be the length of an arc that subtends an angle θ.



A relationship for these three quantities is $\theta = \frac{a}{r}$, where θ is measured in radians.

Radian Measure

We know that there are 360° in a circle, but how many radians is this?

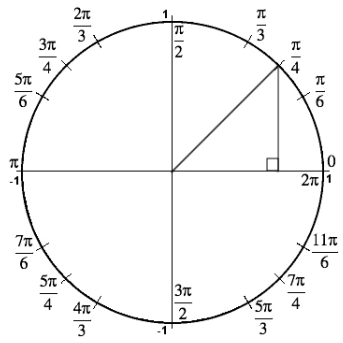
For one full rotation, the arc becomes the circumference.

Since $C = 2\pi r$, this implies that $\theta = \frac{2\pi r}{r} = 2\pi$.

So, there are 2π (approximately 6.28) radians in a circle.

Thus, there are π radians in a semi-circle, $\frac{\pi}{2}$ radians in a quarter-circle, etc.

Radian Measure



Converting Between Radians and Degrees

How many degrees make up one radian, and vice versa?

Set up a proportion as follows:

$$\frac{d^\circ}{1 \text{ rad}} = \frac{360^\circ}{2\pi \text{ rad}}$$
$$d = \frac{360}{2\pi}$$
$$= \frac{180}{\pi}$$

Therefore, one radian is $\frac{180}{\pi}$ degrees.

Using a similar method, one degree is $\frac{\pi}{180}$ radians.

We can use these values to convert between radians and degrees as necessary.

Converting Between Radians and Degrees

Example

Convert 2 rad to degrees.

$2 \times \frac{180}{\pi} \approx 114.6°$.

Example

Convert $\frac{7\pi}{4}$ rad to degrees.

$\frac{7\pi}{4} \times \frac{180}{\pi} = \frac{1260}{4} = 315°$.

Converting Between Radians and Degrees

Example

Convert 50° to radians.

$50 \times \frac{\pi}{180} = \frac{5\pi}{18} \approx 0.873$ rad.

Example

Convert 270° to radians.

$270 \times \frac{\pi}{180} = \frac{3\pi}{2} \approx 4.712$ rad.

When possible, leave all angles in exact form to preserve accuracy.

Arc Length

One of the advantages of using radian measure rather than degrees is that it makes calculating *arc length* on a circle easier.

Recall that one radian is defined as the measure of an angle that is subtended by an arc with a length equal to the circle's radius.

Rearranging the equation $\theta = \frac{a}{r}$ for a, we obtain $a = r\theta$.

This means that if we know a circle's radius and the angle that is formed by the subtended arc, we can calculate the length of the arc.

Arc Length

Example

A bob is at the end of a pendulum with an arm length of 40 cm. If the bob swings through an angle of $\frac{2\pi}{3}$, determine the distance travelled by the bob through the air.

$$\begin{aligned} a &= r\theta \\ &= 40 \times \frac{2\pi}{3} \\ &= \frac{80\pi}{3} \\ &\approx 83.8 \text{ cm} \end{aligned}$$

Arc Length

Example

An arc on the circumference of a circle with a diameter of 12 cm has a length of 22.8 cm. Determine the measure of the angle subtended by the arc.

Since the diameter is 12 cm, the radius is 6 cm.

$$\begin{aligned} \theta &= \frac{22.8}{6} \\ &= 3.8 \text{ rad} \end{aligned}$$

Angular Velocity

As an object rotates, its angular displacement changes with respect to time.

This rate of change is known as an object's *angular velocity*.

Angular Velocity

Given an object's angle of rotation, θ, and time, t, the object's angular velocity, ω, is given by $\omega = \frac{\theta}{t}$.

While it is not strictly necessary to express angular velocity in terms of radians, it typically is.

Angular Velocity

Example

A child rides a carousel that completes 20 revolutions in 2 minutes. Determine the child's angular velocity.

$$\omega = \frac{20 \times 2\pi}{2}$$
$$\omega = 20\pi \text{ rad/min}$$

Questions?