MHF4U: Advanced Functions

Long and Synthetic Division of Polynomials

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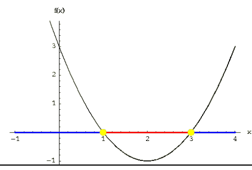

Long Division

Recall...

What is the quotient and remainder when 357 is divided by 8?

$$
\begin{array}{r}
44 \\
8\,\overline{)\,357} \\
\underline{320} \\
37 \\
\underline{32} \\
5
\end{array}
$$

The quotient is 44 and the remainder is 5.

Polynomial Division

Polynomials can be divided by other binomials in the same way that rational numbers are divided by other rationals.

When doing long division with polynomials, each term in the quotient should have a product with the divisor that is equal in value to the current remainder.

It is important to insert terms with zero-coefficients as required.

Polynomial Division

Example

What are the quotient and remainder when $f(x) = x^3 - 2x^2 + 4x - 1$ is divided by $x - 2$.

$$
\begin{array}{r}
x^2 + 0x + 4 \\
x - 2\,\overline{)\,x^3 - 2x^2 + 4x - 1} \\
\underline{-x^3 + 2x^2 \qquad\qquad} \\
0x^2 + 4x \qquad \\
\underline{0x^2 + 0x \qquad} \\
4x - 1 \\
\underline{-4x + 8} \\
7
\end{array}
$$

The quotient is $x^2 + 4$ and the remainder is 7.

Polynomial Division

Example

What are the quotient and remainder when $f(x) = 3x^3 - 5x + 4$ is divided by $x + 3$.

$$
\begin{array}{r}
3x^2 - 9x + 22 \\
x + 3\,\overline{)\,3x^3 + 0x^2 - 5x + 4} \\
\underline{-3x^3 - 9x^2 \qquad\qquad} \\
-9x^2 - 5x \qquad \\
\underline{9x^2 + 27x \qquad} \\
22x + 4 \\
\underline{-22x - 66} \\
-62
\end{array}
$$

The quotient is $3x^2 - 9x + 22$ and the remainder is -62.

Polynomial Division

An alternate way of expressing the quotient and remainder when a polynomial, $P(x)$, is divided by a binomial, $ax - b$, is in the form $P(x) = (ax - b)Q(x) + R$, where $Q(x)$ is the quotient and R is the remainder.

Dividing this through by $ax - b$ results in another form, $\frac{P(x)}{ax - b} = Q(x) + \frac{R}{ax - b}$.

Thus, the last example could be rewritten as either $3x^3 - 5x + 4 = (x + 3)(3x^2 - 9x + 22) - 62$, or as $\frac{3x^3 - 5x + 4}{x + 3} = 3x^2 - 9x + 22 - \frac{62}{x + 3}$.

You should be familiar with these alternate forms, although you may choose to use one of the three to express your answers.

Synthetic Division

Long division can be tedious to write out.

Synthetic division is a shortcut that can be used when a polynomial is divided by a binomial.

It uses the coefficients in a tabular format, saving time.

The root of the binomial is used as a divisor, so a polynomial divided by, say, $x-3$ will use a root of 3.

Synthetic division is best explained via example.

Synthetic Division

Example

Divide $f(x)=x^3-2x^2+4x-1$ by $x-2$.

Since $x-2$ is a factor, use a root of $x=2$.

2	1	-2	4	-1
+		2	0	8
	1	0	4	7

When x^3-2x^2+4x-1 is divided by $x-2$, the quotient is x^2+4 and the remainder is 7.

Therefore, $f(x)=(x-2)(x^2+4)+7$.

Synthetic Division

Example

Divide $g(x)=2x^3+x-3$ by $x+1$.

-1	2	0	1	-3
+		-2	2	-3
	2	-2	3	-6

Therefore, $g(x)=(x+1)(2x^2-2x+3)-6$.

Synthetic Division

Synthetic division works well when a polynomial is divided by a binomial of the form $x-b$.

When the binomial has the form $ax-b$, the quotient will be off by a factor of a, and will need to be divided accordingly.

Synthetic Division

Example

Divide $h(x)=6x^3+x^2-10x+5$ by $3x-1$.

Since $3x-1=0$ has the root $x=\frac{1}{3}$, use this value.

$\frac{1}{3}$	6	1	-10	5
+		2	1	-3
	6	3	-9	2

Dividing the quotient by 3,
$(6x^2+3x-9)\div 3=2x^2+x-3$.

Therefore, $h(x)=(3x-1)(2x^2+x-3)+2$.

Questions?