Equations and Graphs of Polynomial Functions

Polynomial Functions In Factored Form

Example

Identify the factors, and \( x \)-intercepts, of the polynomial function \( f(x) = -2x(x + 5)(x - 3) \).

\( f(x) \) has three factors: \( x \), \( x + 5 \) and \( x - 3 \).

These factors correspond to \( x \)-intercepts 0, -5 and 3.

Order of a Factor

The function \( f(x) = 6x(x - 1)^2(x - 2)^3 \) has \( x \)-intercepts at 0, 1 and 2.

At \( x = 0 \), the function changes from positive to negative, passing through the \( x \)-axis.

The function remains negative on either side of \( x = 1 \), "bouncing" off of the \( x \)-axis.

At \( x = 2 \), the function changes from negative to positive, passing through the \( x \)-axis.

How does this behaviour relate to the factors of the function?

Order of a Factor

In two cases, \( x = 0 \) and \( x = 2 \), the exponents were odd. Both of these cases saw the function change from positive to negative, or vice versa.

In the other case, \( x = 1 \), the exponent was even. No change in sign occurred here.

Order of a Factor

The factor \( (x - r)^n \) has order \( n \). If \( n \) is odd, the function crosses the \( x \)-axis at \( r \). If \( n \) is even, the function touches (but does not cross) the \( x \)-axis at \( r \).

Used in conjunction with a function’s end behaviour, identifying the order of each factor is a useful tool for sketching graphs.
Another way to write the equation is $f(x) = (x+3)(x+1)(x-1)$. Multiplying all terms containing $x$, we obtain $x^3$, so $f(x)$ has degree 3 (cubic).
The leading coefficient is positive, so $f(x)$ has $Q_3$-$Q_1$ end behaviour. Therefore, $f(x)$ is negative as $x \to -\infty$.
Moving from left to right, the first $x$-intercept is at $x = -3$, where it has order 2. Thus, the function touches the $x$-axis at $x = -3$, but stays negative beyond it.
The next $x$-intercept is at $x = 1$, where it has order 1. $f(x)$ changes from negative to positive at $x = 1$.

The next $x$-intercept is at $x = -1$, where it has order 1. $f(x)$ changes from positive to negative at $x = 0$.

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Symmetry

Recall that an even function is symmetric in the $f(x)$-axis.

Any function that is symmetric in the $f(x)$-axis has the property that $f(x) = f(-x)$.

An odd function is point-symmetric about the origin.

Any function that has point symmetry about the origin has the property that $f(-x) = -f(x)$.

Example

Verify algebraically that $f(x) = 2x^4 + 3x^2 - 1$ is symmetric in the $f(x)$-axis.

$$f(-x) = 2(-x)^4 + 3(-x)^2 - 1$$
$$= 2x^4 + 3x^2 - 1$$
$$= f(x)$$

Therefore, $f(x)$ is symmetric in the $f(x)$-axis. It is an even function.

Example

Algebraically classify $f(x) = 2x^3 + x^2 - 5x$ as even, odd or neither.

Test if $f(x)$ is even first.

$$f(-x) = 2(-x)^3 + (-x)^2 - 5(-x)$$
$$= -2x^3 + x^2 + 5x$$
$$\neq f(x)$$

Therefore, $f(x)$ is not even.

Test if $f(x)$ is odd next.

$$f(-x) = -2x^3 + x^2 + 5x$$
$$- f(x) = -(2x^3 + x^2 - 5x)$$
$$= -2x^3 - x^2 + 5x$$
$$f(-x) \neq -f(x)$$

Therefore, $f(x)$ is not odd either.

Questions?