

# MHF4U Final Examination Information

The final examination is split into two components, each worth 15% of your final mark. You *must* be present on the day of your examination, or provide medical documentation that states “[your name] was unable to write his/her/their examination on the date indicated” otherwise.

## In-Class Component: Thursday, January 9, 2020 (75 min)

- Polynomial functions and inequalities (chap 1-2)
- Rational functions and inequalities (chap 3)

## Exam Period Component: See Exam Schedule (2 hr)

- Trigonometric functions and equations (chap 4/5)
- Trigonometric identities (chap 4/5)
- Exponential and logarithmic functions and equations (chap 6-7)
- Composition of functions (chap 8)

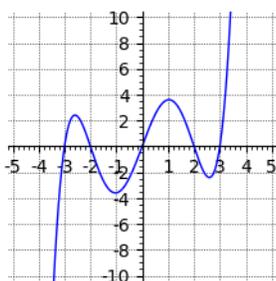
## Examination Format

Each component consists of multiple choice and full solution questions. Multiple choice questions assess understanding of key concepts, or the ability to connect general ideas to various representations of functions and procedures. They may involve *short* calculations. They should *not* take a long time to answer. If you find yourself performing 5 minutes of calculations for a multiple choice question, you are likely over-complicating the situation. A sample is below.

### Sample Question

1. Which of the following represents an even polynomial function?

A. The graph below.



B. The equation  
 $f(x) = x^4 - 5x^2 + x - 1$ .

C. The equation  
 $g(x) = 5x^2(x-3)(x+3)$ .

D. The table below.

x	y		
0	0	$\Delta 1$	
1	...	...	$\Delta 2$
2	...	...	$\Delta 3$
3	...	...	9
4	...	...	9
5	...	...	9

### Solution and Rationale

C is an even function. Even functions have line symmetry in the y-axis, which the graph in A does not. While B is a quartic function, all of the exponents are not even integers, so B cannot be even. The constant third differences indicate that the polynomial in D a cubic function, so it cannot be even. C is a quartic function with roots at 0 (order 2), 3 and -3. Since these are symmetric, C must be an even function.

□

Full solution questions assess several things: an understanding of fundamental concepts; the ability to interpret a question; the capacity to develop a procedure to solve a problem; the ability to select and apply appropriate tools; and the use of proper mathematical style and form. They are not assessed solely on correct answers. To achieve a level 3 or 4, all relevant steps must be included. These may take the form of calculations, graphs, or logical explanations using proper terminology. Final answers should be clearly indicated. A sample is below.

Sample Question

Determine the value(s) of the roots for the cubic polynomial that produces the following table of values.

x	y			
0	216	$\Delta 1$		
1	192	-24	$\Delta 2$	
2	140	-52	-28	$\Delta 3$
3	72	-68	-16	12
4	0	-72	-4	12
5	-64	-64	8	12

Solution

A cubic polynomial has a general equation  $f(x)=ax^3+bx^2+cx+d$ . Since the third differences have a value of 12, the leading coefficient is  $a=\frac{12}{3!}=2$ . The table of values also indicate a y-intercept at 216, so  $d=216$ . This gives us the equation  $f(x)=2x^3+bx^2+cx+216$ . Substitute two other points into this equation to obtain two new ones.

$$\begin{aligned} 2(1)^3+b(1)^2+c(1)+216 &= 192 & 2(2)^3+b(2)^2+c(2)+216 &= 140 \\ 2+b+c+216 &= 192 & \text{and} & 16+4b+2c+216 &= 140 \\ b+c &= -26 & & 4b+2c &= -92 \end{aligned}$$

Use elimination (or substitution) to solve for  $b$  and  $c$ .

$$\begin{aligned} b+c &= -26 & \Rightarrow & & 2b+2c &= -52 \\ 4b+2c &= -92 & - & 4b+2c &= -92 \\ & & & & -2b &= 40 \\ & & & & b &= -20 \end{aligned}$$

Since  $b+c=-26$ ,  $c=-6$  when  $b=-20$ . This gives the equation  $f(x)=2x^3-20x^2-6x+216$ . From the table of values, we know that the polynomial has a root when  $x$  is 4. Use synthetic division to determine the other roots.

$$\begin{array}{r|rrrrr} 4 & 2 & -20 & -6 & 216 & \\ + & \downarrow & & 8 & -48 & 216 \\ \times & 2 & -12 & -54 & 0 & \end{array}$$

The quotient is  $2x^2-12x-54=2(x^2-6x-27)=2(x+3)(x-9)$ , so the cubic polynomial has an equation of  $f(x)=2(x-4)(x+3)(x-9)$  in factored form. Thus, the roots are 4, -3 and 9.

□