MCV4U: Calculus & Vectors

Resolving Vectors Into Components

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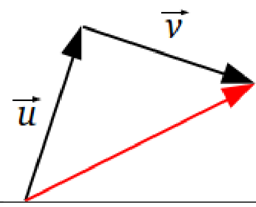


Resolving Vectors Into Components

A vector, $\vec{v}$, can be *resolved* into its vertical and horizontal components, $\vec{v_v}$ and $\vec{v_h}$.

These two vectors, called the *rectangular components*, are perpendicular and their sum is $\vec{v}$.

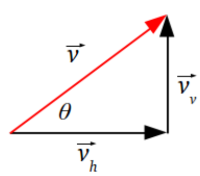
We can use trigonometry to determine the magnitudes of these components.

Resolving Vectors Into Components

Rectangular Components of a Vector

For any vector $\vec{v}$ at an angle of θ to the horizontal:

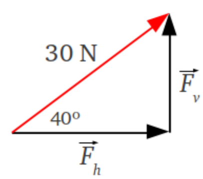
- the magnitude of the vertical component is given by $|\vec{v_v}| = |\vec{v}| \sin\theta$
- the magnitude of the horizontal component is given by $|\vec{v_h}| = |\vec{v}| \cos\theta$



Resolving Vectors Into Components

Example

A child pulls a wagon with a 30 N force, at an angle of 40° to the horizontal. Determine the magnitudes of the horizontal and vertical components



Resolving Vectors Into Components

For the vertical component, $|\vec{F_v}| = 30 \sin 40° \approx 19.28$ N.

For the horizontal component, $|\vec{F_h}| = 30 \cos 40° \approx 22.98$ N.

We can confirm our calculation using the Pythagorean theorem.

$$\begin{aligned}(30\sin 40°)^2 + (30\cos 40°)^2 &= 900\sin^2 40° + 900\cos^2 40° \\ &= 900(\sin^2 40° + \cos^2 40°) \\ &= 900(1) \\ &= 30^2\end{aligned}$$

Non-Vertical/Horizontal Components

Example

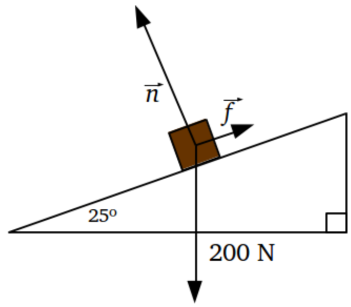
A crate weighing 200N is at rest on a ramp, inclined at an angle of 25°. Resolve the weight into its rectangular components.

The box is in equilibrium, since it is at rest.

There are three forces acting upon the crate:

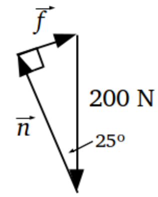
- the force of gravity acting downward, for a total force of 200 N
- the *normal force*, perpendicular to the surface of the ramp
- the force of *friction*, parallel to the surface of the ramp

Non-Vertical/Horizontal Components



Non-Vertical/Horizontal Components

Redraw the diagram, such that the three vectors sum to the zero vector, since the crate is at rest.



Non-Vertical/Horizontal Components

Let $\vec{f}$ be the force of friction, and let $\vec{n}$ be the normal force.

Since $\vec{f}$ and $\vec{n}$ are perpendicular, treat them as the vertical and horizontal components.

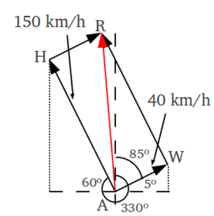
$|\vec{f}| = 200 \sin 25° \approx 84.5$ N.

$|\vec{n}| = 200 \cos 25° \approx 181.3$ N.

Finding a Resultant Using Components

Example

An airplane is travelling on a bearing of 330° at a constant speed of 150 km/h. A wind blows on a bearing of 85° at 40 km/h. Determine the ground speed, and direction, of the airplane.



Finding a Resultant Using Components

For convenience, define North and East as positive, and South and West as negative.

Break down the vectors representing the wind, $\vec{w}$, and the airplane, $\vec{h}$, into their components.

$\vec{w_x} = 40 \cos 5° \approx 39.85$ km/h.

$\vec{w_y} = 40 \sin 5° \approx 3.49$ km/h.

$\vec{h_x} = -150 \cos 60° \approx -75$ km/h.

$\vec{h_y} = 150 \sin 60° \approx 129.90$ km/h.

Finding a Resultant Using Components

Add the horizontal components, then add the vertical components.

$|\vec{R_x}| \approx 39.85 + (-75) \approx -35.15$ km/h.

$|\vec{R_y}| \approx 3.49 + 129.90 \approx 133.39$ km/h.

Use the Pythagorean theorem to find the resultant's magnitude, and the tangent ratio to find its angle.

$$|\vec{R}| \approx \sqrt{(-35.15)^2 + 133.39^2} \approx 138 \text{ km/h} \qquad \theta \approx \tan^{-1}\left(\frac{35.15}{133.39}\right) \approx 15°$$

The airplane has a ground speed of approximately 138 km/h on a bearing of 345°.

Using Components With Multiple Forces

When many forces act upon an object, using components may be the best method.

For example, assume that four forces are acting on an object. First, the resultant of two forces would have to be determined using the cosine law.

This resultant would then need to be used with a third force (using the cosine law again) to produce a new resultant.

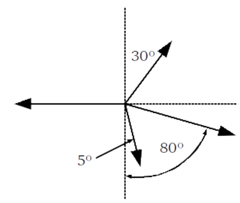
Finally, this new resultant would need to be used with the fourth force (cosine law again) to produce the final resultant.

Using Components With Multiple Forces

Example

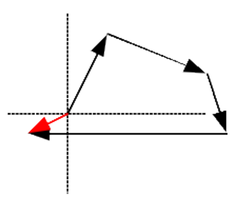
Four forces act on an object: 16 N [N30°E], 25 N [S80°E], 12 N [S5°E] and 40 N [W]. Determine the magnitude and direction of the resulting force.

Draw a diagram to visualize the forces.



Using Components With Multiple Forces

Arranging the vectors tip-to-tail gives a more accurate picture of the resultant.



The resultant is small, and directed somewhere southwest.

Using Components With Multiple Forces

Break down each force into its components.

$$\vec{F1}_x = 16\cos 60^\circ = 8 \text{ N} \qquad \vec{F2}_x = 25\cos 10^\circ \approx 24.62 \text{ N}$$

$$\vec{F3}_x = 12\cos 85^\circ \approx 1.05 \qquad \vec{F4}_x = -40 \text{ N}$$

$$\vec{F1}_y = 16\sin 60^\circ \approx 13.86 \text{ N} \qquad \vec{F2}_y = -25\sin 10^\circ \approx -4.34 \text{ N}$$

$$\vec{F3}_y = -12\sin 85^\circ \approx -11.95 \text{ N} \qquad \vec{F4}_y = 0 \text{ N}$$

Using Components With Multiple Forces

Add the horizontal components, then add the vertical components.

$|\vec{R_x}| \approx 8 + 24.62 + 1.05 - 40 \approx -6.35$ N.

$|\vec{R_y}| \approx 13.86 - 4.34 - 11.95 + 0 \approx -2.43$ N.

Use the Pythagorean theorem to find the resultant's magnitude, and the tangent ratio to find its angle.

$$|\vec{R}| \approx \sqrt{6.35^2 + 2.43^2} \approx 6.8 \text{ N} \qquad \theta \approx \tan^{-1}\left(\frac{6.35}{2.43}\right) \approx 69^\circ$$

The resultant force is approximately 6.8 N [S69°W].

Questions?