MCV4U: Calculus & Vectors

Sketching Planes

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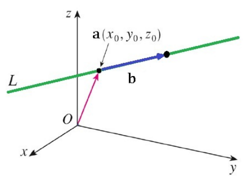


Sketching Planes

Visualizing a plane is sometimes useful for verifying answers, or for determining how planes are arranged in R^3 when solving more complex problems.

Sketches are only as accurate as you make them, and may not be entirely helpful for all situations.

When analyzing equations of planes, look for key features that will help, such as parallelism or perpendicularity, or passing through key points.

Sketching Planes

Example

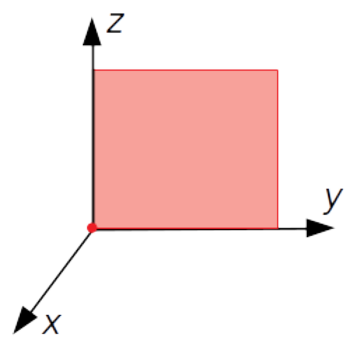
Sketch the plane $x = 0$.

In the equation, both y and z are omitted, so *any* values of y and z are permissible.

At the same time, x is always fixed at 0. Therefore, the plane must pass through $(0, 0, 0)$.

Therefore, the equation $x = 0$ represents the plane that makes up the yz-plane.

Sketching Planes



Sketching Planes

Example

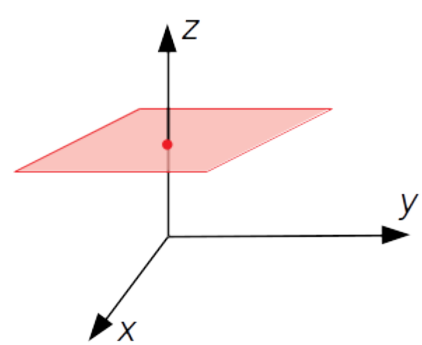
Sketch the plane $z = 5$.

Like before, two variables (x and y) are omitted from the equation, so they can assume any values.

Since z is always 5, the plane does not pass through the origin.

Therefore, the equation $z = 5$ represents a plane that is parallel to the xy-plane, but 5 units away through $(0, 0, 5)$.

Sketching Planes



Sketching Planes

Example

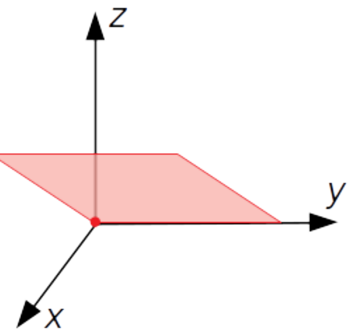
Sketch the plane $2x - z = 0$.

Since y is omitted from the equation, it can be any value.

Since there is no constant in the equation, substituting $x = 0$ and $z = 0$ gives us all points of the form $(0, y, 0)$. Thus, the plane contains the y-axis.

To visualize, select any point in the xz-plane that satisfies the equation, such as $(1, 0, 2)$, and represent the plane as a parallelogram.

Sketching Planes



Sketching Planes

Example

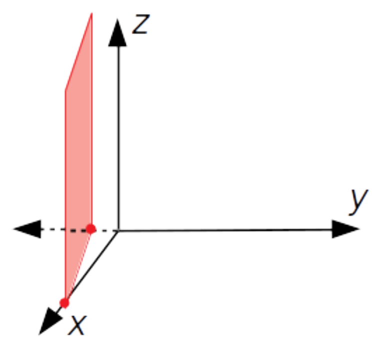
Sketch the plane $2x - 5y - 10 = 0$.

Since z is omitted from the equation, it can be any value.

Due to the constant, substituting $x = 0$ and $z = 0$ does not work, since $2(0) - 5(0) - 10 \neq 0$. Thus, the plane does *not* contain the z-axis. It *will*, however, be parallel to the z-axis.

A possible way to visualize is to determine the two other intercepts, and use them to sketch a parallelogram. In this case, the intercepts are at $(5, 0, 0)$ and $(0, -2, 0)$.

Sketching Planes



Sketching Planes

Example

Sketch the plane $x + 3y - z = 0$.

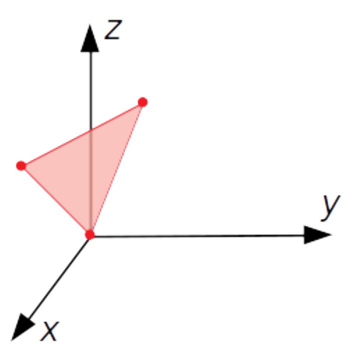
Since $x = 0$, $y = 0$, $z = 0$ satisfies the equation, the plane passes through the origin.

As all of the intercepts are at this point, we need to find additional points to visualize the plane. Using small values like 0 and 1 might be useful.

Instead of a parallelogram, we can use a triangle to represent a plane. This cuts down on the number of points required.

In this case, two additional points are $(1, 0, 1)$ and $(0, 1, 3)$.

Sketching Planes



Sketching Planes

Example

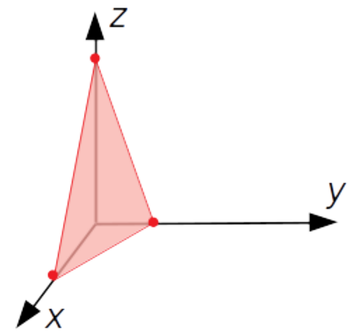
Sketch the plane $2x + 3y + z - 6 = 0$.

Due to the constant, the plane does not pass through the origin.

Find three points to visualize the plane as a triangle. Intercepts are easy to find.

Three points on the plane are $(3, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 6)$.

Sketching Planes



Questions?