MCV4U: Calculus & Vectors

Basic Derivative Rules

(Constant, Power, Multiple, Sum and Difference)

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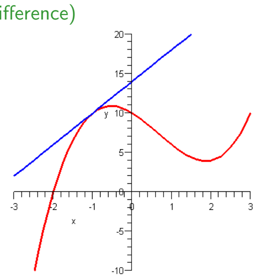

Basic Derivative Rules

While we have a method that uses limits to find the derivative of a function, there are many rules that can help speed up the process.

In particular, polynomial functions have easy-to-remember rules that allow us to avoid any work with limits entirely.

Other functions, such as reciprocals or those that involve radicals, can generally be solved using the same techniques.

Basic Derivative Rules

Constant Rule

If $f(x) = c$, where c is a constant, then $f'(x) = 0$.

If $y = c$, where c is some real constant, then $\frac{dy}{dx} = 0$.

Geometrically, the function $f(x) = c$ is a horizontal line with a slope of zero. Therefore, the slope of the tangent to any point on the line will also be zero.

More formally, we can use the limit definition of the derivative to arrive at the same result.

Basic Derivative Rules

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Since the function is a horizontal line, $f(x+h) = f(x)$ for any value of h.

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{0}{h} \\ &= 0 \end{aligned}$$

Remember that $h \to 0$ means that h gets *infinitesimally close* to zero, but does not have to *be* zero. Thus, there is no division-by-zero issue.

Basic Derivative Rules

Examples

Determine the derivatives of $f(x) = 10$ and $g(x) = 3\left(\frac{\pi}{e}\right)^2$.

$$\begin{aligned} f'(x) &= 0 \\ g'(x) &= 0 \end{aligned}$$

Remember that π and e are constants!

Basic Derivative Rules

Power Rule

If $f(x) = x^n$, where n is a real number, then $f'(x) = nx^{n-1}$.

If $y = x^n$, where n is a real number, then $\frac{dy}{dx} = nx^{n-1}$.

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

Note that a difference of powers $a^n - b^n$ can be factored as $(a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + ab^{n-2} + b^{n-1})$.

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{[x+h-x][(x+h)^{n-1} + (x+h)^{n-2}x + \ldots + x^{n-1}]}{h} \\ &= \lim_{h \to 0} \underbrace{[(x+h)^{n-1} + (x+h)^{n-2}x + \ldots + x^{n-1}]}_{n \text{ terms}} \end{aligned}$$

Basic Derivative Rules

As $h \to 0$, each term containing $(x+h)$ becomes a term containing only x.

$$f'(x) = \underbrace{x^{n-1} + x^{n-2}x + \ldots + x^{n-1}}_{n \text{ terms}}$$

Use exponent laws to simplify.

$$f'(x) = \underbrace{x^{n-1} + x^{n-1} + \ldots + x^{n-1}}_{n \text{ terms}}$$
$$= nx^{n-1}$$

Basic Derivative Rules

Examples

Determine the derivatives of $f(x) = x^{10}$ and $g(t) = \frac{1}{t^5}$.

$$f'(x) = 10x^{10-1}$$
$$= 10x^9$$

$$g(t) = t^{-5}$$
$$g'(t) = -5t^{-6} \text{ or } -\frac{5}{t^6}$$

Basic Derivative Rules

Constant Multiple Rule

If $f(x) = kg(x)$ for some real value k, then $f'(x) = kg'(x)$.

If $y = ku$ for some real value k, then $\frac{dy}{dx} = k\frac{du}{dx}$.

$$f'(x) = \lim_{h\to0} \frac{kg(x+h) - kg(x)}{h}$$

Common factor k and apply properties of limits.

$$f'(x) = \lim_{h\to0} k\left(\frac{g(x+h)-g(x)}{h}\right)$$
$$f'(x) = k\lim_{h\to0} \left(\frac{g(x+h)-g(x)}{h}\right)$$
$$f'(x) = kg'(x)$$

Basic Derivative Rules

Examples

Determine the derivatives of $f(x) = 5x^3$ and $g(a) = \frac{6}{a^2}$.

$$f'(x) = 5 \times (x^3)'$$
$$= 5 \times (3x^2)$$
$$= 15x^2$$

$$g(x) = 6a^{-2}$$
$$g'(x) = -12a^{-3} \text{ or } -\frac{12}{a^3}$$

Basic Derivative Rules

Sum and Difference Rules

If $p(x)$ and $q(x)$ are both differentiable and $f(x) = p(x) \pm q(x)$, then $f'(x) = p'(x) \pm q'(x)$.

If $y = u \pm v$, then $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$.

$$f'(x) = \lim_{h\to0} \frac{[p(x+h) \pm q(x+h)] - [p(x) \pm q(x)]}{h}$$
$$= \lim_{h\to0} \left(\frac{p(x+h)-p(x)}{h} \pm \frac{q(x+h)-q(x)}{h}\right)$$
$$= \lim_{h\to0} \frac{p(x+h)-p(x)}{h} \pm \lim_{h\to0} \frac{q(x+h)-q(x)}{h}$$
$$= p'(x) \pm q'(x)$$

Basic Derivative Rules

Examples

Determine the derivatives of $f(x) = 3x^4 - 5$ and $g(n) = -2n^3 - 4\sqrt{n}$.

$$f'(x) = (4 \cdot 3)x^{4-1} - 0$$
$$= 12x^3$$

$$g(n) = -2n^3 - 4n^{\frac{1}{2}}$$
$$g'(n) = -6n^2 - 2n^{-\frac{1}{2}} \text{ or } -6n^2 - \frac{2}{\sqrt{n}}$$

Applying the Rules

Example

Determine the equation of the tangent to $f(x) = 6\sqrt{x} + 1$ when $x = 4$.

When $x = 4$, $f(x) = 13$, so the point of tangency is $(4, 13)$.

Determine the derivative and evaluate at $x = 4$ to find the slope of the tangent.

$$\begin{aligned} f(x) &= 6x^{\frac{1}{2}} + 1 \\ f'(x) &= 3x^{-\frac{1}{2}} \\ f'(4) &= \tfrac{3}{2} \end{aligned}$$

Applying the Rules

Solve for the equation of the line.

$$\begin{aligned} 13 &= \tfrac{3}{2}(4) + b \\ b &= 7 \end{aligned}$$

The equation of the tangent to $f(x) = 6\sqrt{x} + 1$ when $x = 4$ is $y = \frac{3}{2}x + 7$.

Applying the Rules

Example

Determine any points on the graph of $y = 2x^4 + 5x^2 - 1$ where the tangent has a slope of 6.

Find $\frac{dy}{dx}$, which represents the slope of the tangent.

$$\frac{dy}{dx} = 8x^3 + 10x$$

Set $\frac{dy}{dx} = 6$ and solve for x.

$$\begin{aligned} 8x^3 + 10x &= 6 \\ 8x^3 + 10x - 6 &= 0 \end{aligned}$$

Applying the Rules

Since $8\left(\frac{1}{2}\right)^3 + 10\left(\frac{1}{2}\right) - 6 = 0$, $(2x - 1)$ is a factor.

	8	0	10	−6
$\frac{1}{2}$		4	2	6
	8	4	12	0

Thus, $y = (2x - 1)(4x^2 + 2x + 6)$. Remember to divide the quotient by the denominator.

The quadratic expression $4x^2 + 2x + 6$ has no real roots, since $2^2 - 4(4)(6) < 0$.

Therefore, the only point on the graph of $y = 2x^4 + 5x^2 - 1$ whose tangent has a slope of 6 occurs when $x = \frac{1}{2}$.

Since $2\left(\frac{1}{2}\right)^4 + 5\left(\frac{1}{2}\right)^2 - 1 = \frac{3}{8}$, the point is $\left(\frac{1}{2}, \frac{3}{8}\right)$.

Questions?