MCV4U: Calculus & Vectors

Adding and Subtracting Vectors

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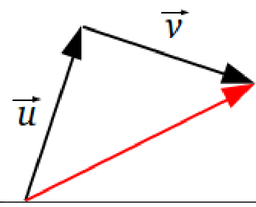


Methods of Adding Vectors Geometrically

Recall that two vectors are equivalent if they have the same magnitude and direction.

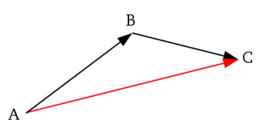
This means that vectors can change their positions and remain equivalent, as long as they maintain their magnitudes and directions.

This makes it possible for us to construct diagrams that represent vector addition or subtraction of two or more vectors.

Methods of Adding Vectors Geometrically

Triangle Method of Vector Addition

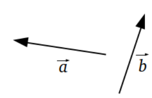
Given two vectors, $\vec{AB}$ and $\vec{BC}$, arranged head to tail as shown below, the resultant $\vec{AC}$ is the sum of $\vec{AB} + \vec{BC}$.



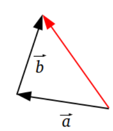
Methods of Adding Vectors Geometrically

Example

Given vectors $\vec{a}$ and $\vec{b}$, draw $\vec{a} + \vec{b}$.



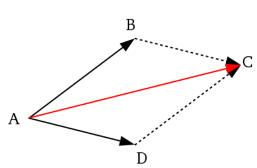
Using the triangle method of vector addition,



Methods of Adding Vectors Geometrically

Parallelogram Method of Vector Addition

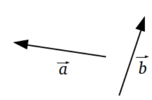
Given two vectors, $\vec{AB}$ and $\vec{AD}$, arranged tail-to-tail as shown, let $\vec{BC} = \vec{AD}$ and $\vec{DC} = \vec{AB}$. The resultant $\vec{AC}$ is the sum of $\vec{AB} + \vec{BC}$ or $\vec{AD} + \vec{DC}$.



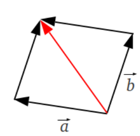
Methods of Adding Vectors Geometrically

Example

Given vectors $\vec{a}$ and $\vec{b}$, draw $\vec{a} + \vec{b}$.



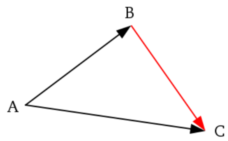
Using the parallelogram method of vector addition,



Methods of Subtracting Vectors Geometrically

Tail-to-Tail Method of Vector Subtraction

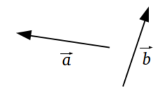
Given two vectors, $\vec{AB}$ and $\vec{AC}$, arranged tail-to-tail as shown, the resultant $\vec{BC}$ is the difference of $\vec{AC} - \vec{AB}$.



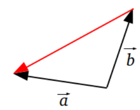
Methods of Subtracting Vectors Geometrically

Example

Given vectors $\vec{a}$ and $\vec{b}$, draw $\vec{a} - \vec{b}$.



Using the tail-to-tail method of vector subtraction,

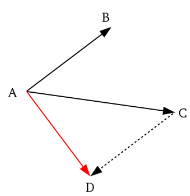


Methods of Subtracting Vectors Geometrically

Alternatively, a vector may be subtracted from another using its opposite vector.

Opposite Vector Method of Vector Subtraction

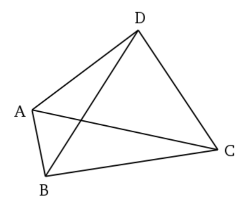
Given two vectors, $\vec{AB}$ and $\vec{AC}$, arranged tail to tail as shown, let $\vec{CD} = -\vec{AB} = \vec{BA}$. The resultant $\vec{AD}$ is the difference of $\vec{AC} - \vec{AB}$.



Adding and Subtracting Vectors

Example

Using the following diagram, express $\vec{AB} + \vec{BC}$ as a single vector.

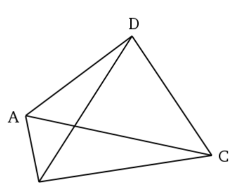


$\vec{AB} + \vec{BC} = \vec{AC}$

Adding and Subtracting Vectors

Example

Using the following diagram, express $\vec{DB} - \vec{CB}$ as a single vector.

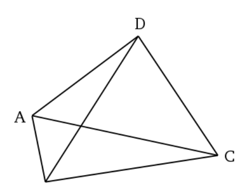


$\vec{DB} - \vec{CB} = \vec{DB} + \vec{BC} = \vec{DC}$

Adding and Subtracting Vectors

Example

Using the following diagram, express $(\vec{BC} + \vec{CD}) + \vec{DA}$ as a single vector.

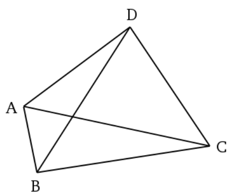


$(\vec{BC} + \vec{CD}) + \vec{DA} = \vec{BD} + \vec{DA} = \vec{BA}$

Adding and Subtracting Vectors

Example

Using the following diagram, express $\vec{BC} + (\vec{CD} + \vec{DA})$ as a single vector.



$\vec{BC} + (\vec{CD} + \vec{DA}) = \vec{BC} + \vec{CA} = \vec{BA}$

Adding and Subtracting Vectors

The last example illustrates the *associative property* of vector addition.

Properties of Vector Addition and Subtraction

Given vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$:

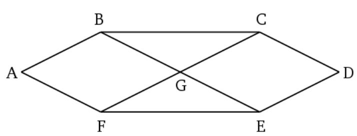
- $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ (associative property)
- $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ (commutative property)
- $\vec{v} + \vec{0} = \vec{v}$ (identity property)

The *zero vector*, $\vec{0}$, has a magnitude of zero and arbitrary direction. Thus, adding a vector to the zero vector results in the original vector.

Adding and Subtracting Vectors

Example

Using the following diagram, let $\vec{AB} = \vec{x}$ and $\vec{BC} = \vec{y}$. Express $\vec{EF}$ in terms of $\vec{x}$ and $\vec{y}$.

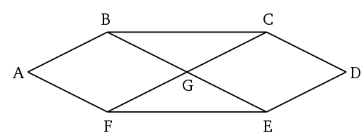


$\vec{EF} = \vec{CB} = -\vec{BC} = -\vec{y}$

Adding and Subtracting Vectors

Example

Using the following diagram, let $\vec{AB} = \vec{x}$ and $\vec{BC} = \vec{y}$. Express $\vec{BG}$ in terms of $\vec{x}$ and $\vec{y}$.

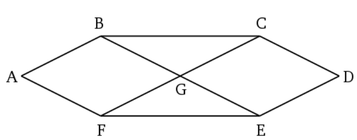


$\vec{BG} = \vec{BC} + \vec{CG} = \vec{BC} + \vec{BA} = \vec{BC} - \vec{AB} = \vec{y} - \vec{x}$

Adding and Subtracting Vectors

Example

Using the following diagram, let $\vec{AB} = \vec{x}$ and $\vec{BC} = \vec{y}$. Express $\vec{AD}$ in terms of $\vec{x}$ and $\vec{y}$.



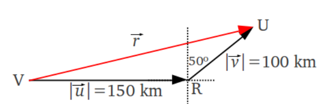
$\vec{AD} = \vec{AB} + \vec{BC} + \vec{CD} = \vec{AB} + \vec{BC} + \vec{BG} = \vec{x} + \vec{y} + \vec{y} - \vec{x} = 2\vec{y}$

Adding and Subtracting Vectors

Example

A ship travels 150 km due east of port, then assumes a bearing of N50°E for 100 km. Use trigonometry to determine the displacement of the ship, and its direction.

Use the following diagram.



Adding and Subtracting Vectors

The displacement is $|\vec{r}|$, where r is the resultant vector. Use the cosine law.

$$\begin{aligned} |\vec{r}| &= \sqrt{|\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}|\cos R} \\ &= \sqrt{150^2 + 100^2 - 2 \cdot 150 \cdot 100 \cos 140^\circ} \\ &\approx 235.5km \end{aligned}$$

Adding and Subtracting Vectors

The direction can be found if we know the measure of $\angle V$. Use the sine law.

$$\begin{aligned} \frac{\sin V}{|\vec{v}|} &= \frac{\sin R}{|\vec{r}|} \\ \angle V &\approx sin^{-1}\left(\frac{100 \cdot \sin 140^\circ}{235.5}\right) \\ &\approx 16^\circ \end{aligned}$$

The displacement is approximately 235.5 km, at a bearing of approximately N74°E.

Questions?