MCV 4U0 REVIEW FOR FINAL EXAMINATION

PART A: OVERALL REVIEW

1. Find the derivative of each of the following by first principles.

a) $5x^2+3x$ b) $\sqrt{x^2+1}$ c) $\sin x$

2. Evaluate each limit.

a) $\lim_{x\to -1}(2x+5)$ b) $\lim_{x\to -3}\frac{x^2-9}{x+3}$ c) $\lim_{x\to 1}\frac{1-\sqrt{x}}{1-x}$ d) $\lim_{x\to 0}\frac{\sqrt{1-x}-1}{x}$

e) $\lim_{x\to 27}\frac{x^{\frac{1}{3}}-3}{x-27}$ f) $\lim_{x\to\infty}\frac{2x^2-5}{3x^2-x+7}$ g) $\lim_{x\to\infty}\frac{5-6x+3x^2}{8-9x+x^3}$ h) $\lim_{x\to -3^-}\frac{x}{(x+3)(x-2)}$

i) $\lim_{x\to\sqrt{3}^+}\frac{x-2}{x^2-3}$ j) $\lim_{h\to 0}\frac{\sin 5h}{h}$ k) $\lim_{x\to 0}\frac{\tan x-\sin x}{x\cos x}$ l) $\lim_{m\to\infty}\left(1+\frac{1}{m-50}\right)^m$

3. Find the derivative of each of the following.

a) $(3x+4)^3(2x^2-1)^2$ b) $(3x^2-5)^7$ c) $\left(\frac{2x+1}{2x-1}\right)^3$ d) $3x^2-xy+2y^3=7$

e) $\frac{x^2-6}{\sqrt{x}}$ f) $y=\tan^4 3x$ g) $y=\frac{\cos 5x}{\sin 5x}$ h) $y=\sin^2(5x-1)$

i) $f(x)=\ln(x^2+1)$ j) $f(x)=e^{3x^2}$ k) $y=\ln x^6$ l) $y=\ln(\sin x)$

m) $y=\ln\left(\frac{3x^2-7}{2x+1}\right)$ n) $\ln\sqrt{\frac{2x-1}{8x+3}}$ o) $y=7x^2e^{5x+1}$ p) $y=e^x\ln x$

q) $y=\frac{e^x}{x^2}$ r) $\log_{10}(x^2+3x)$ s) $y=\cot\left(\ln\left(\sqrt{\frac{3}{e^5}}\right)\right)$ t) $y=4^{x^2}$

u) $y=\sin^{-1}(5x)$ v) $y=(\sin x)^{x^2}$

4. Find the equation of the tangent line and the normal line to the curve at the given point.

a) $y=x^2-2x+5$ at $(-1,8)$ b) $y=\frac{2}{1-x}$, $(2,-2)$ c) $x^3+y^3=9xy$, $(2,4)$

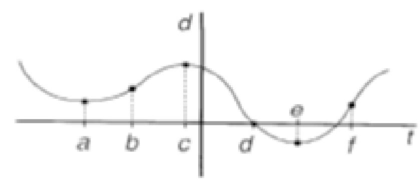
d) $y=x\sin x$, $\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ e) $y=\frac{e^x}{1+\ln x}$ when $x=1$

5. At what points on the curve $y=\sin x+\cos x$, $0\le x\le 2\pi$, is the tangent line horizontal?

Part B: Distance, Velocity & Acceleration

1. On a windless day, a foolish archer shoots an arrow vertically upward so that its height in metres above the ground t seconds after release is given by the formula $h(t) = -4.9t^2 + 24.5t + 2$
   a) At what time does the arrow stop rising?
   b) How many seconds does the archer have before he risks serious personal injury by not moving? (The archer is 2 m tall.)
   c) Show that the acceleration of the arrow is a constant. Why does the acceleration not vary with time?

2. The following graph represents the distance of a particle from a fixed point over time. Determine the intervals for which
   a) both the distance and the velocity are increasing
   b) the distance is increasing while the velocity is decreasing
   c) the distance is decreasing while the velocity is increasing
   d) both the distance and the velocity are decreasing.



3. The position function describes the motion of an object that accelerates from rest at $t = 0$ and subsequently brakes and comes to a stop. Find the total distance travelled and the distance travelled before braking begins. $s(t) = t^2(10-t)^3$

Part C: Optimization

1. At 15:00 hours, a merchant ship sailing south at 18 km/h is 40 nautical km due east of a patrol boat travelling east at 24 km/h. When will they be closest to each other?

2. A box with a square base and open top must have a volume of 4000 cm^3. Find the dimensions of the box that minimizes the amount of material used.

3. A box with an open top is to be constructed from a square piece of cardboard, 6m wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

5. The cross section of an eavestrough has the shape of a triangle, in which the two sides are 4 cm in width, determine the width across the top of the trough which maximizes the area of the cross section.

4. The lifeguard at a public beach has 400 m of rope available to lay out a rectangular restricted swimming area using the straight shoreline as one side of the rectangle.
   a) If she wants to maximize the swimming area, what will the dimensions of the rectangle be?

   b) To ensure the safety of swimmers, she decides that nobody should be more than 50 m from shore. What should the dimensions of the swimming area be with this added restriction?

PART D: MAXIMUM & MINIMUM VALUES

1. Find the absolute maximum value and absolute minimum value of the function.
   a) $f(x) = 2x^3 - 3x^2, \quad -2 \le x \le 2$ b) $\cos^2 x - \cos x = 0, \quad 0 \le x \le 2\pi$
2. The position of an object that moves in a straight line is $s(t) = 1 + 2t - \dfrac{8}{t^2+1}, \quad 0 \le t \le 2$.
   Find the maximum and minimum velocities of the object over the given time intervals.
3. The motion of a particle is described by the equation $s(t) = 10\cos(5t - \pi/4)$. What are the maximum values of the displacement, the velocity, and the acceleration? At what times t do these maxima occur? What are the minimum values of these quantities?

PART E: CURVE SKETCHING

1. Find the vertical and horizontal asymptotes.
   a) $y = 6x^2 - 2$ b) $y = \dfrac{x-3}{x^2-9}$ c) $f(x) = \dfrac{x^3-8}{x^2+7x+12}$
2. Find the interval on which the curve is concave upwards or concave downward and state the points of inflection.
   a) $f(x) = 5x^3 + 12x^2 - 3x + 2$ b) $f(x) = 2x^3 + 15x^2 - 36x$
3. Find the local maximum and minimum values of each function.
   a) $y = x^3 - 6x^2 + 9x$ b) $f(x) = x^2 - x$
4. Sketch the following curves.
   a) $y = 2 - 12x + 9x^2 - 2x^3$. b) $y = \dfrac{1+x^2}{1-x^2}$

Answers

Part A: Overall Review

1. a) $10x+3$ b) $x/\sqrt{x^2+1}$ c) $\cos x$
2. a) 3 b) -6 c) $1/2$ d) $-1/2$ e) 1/27 f) 2/3 g) 0 h) $+\infty$ i) $+\infty$ j) 5 k) 0 l) e
3. a) $(2x^2-1)(3x+4)^2(42x^2+32x-9)$ b) $42x(3x^2-6)^6$ c) $-12(2x+1)^2/(2x-1)^4$ d) $(y-6x)/(6y^2-x)$ e) $(3x^2+6)/(2x\sqrt{x})$

f) $12\tan^3 3x\sec^2 3x$ g) $-5\csc^2 5x$ h) $5(2\sin(5x-1)\cos(5x-1))=5\sin 2(5x-1)$ i) $2x/(x^2+1)$ j) $6xe^{3x^2}$ k) 6/x l) $\tan x$

m) $6x/(3x^2-7)-2/(2x+1)$ n) $1/(2x-1)-4/(8x+3)$ o) $e^{5x+1}(14x+35x^2)$ p) $e^x(\ln x+1/x)$ q) $e^x(x-2)/x^3$

n) r) $(2x+3)/\ln 10(x^2+3x)$ s) 0 t) $4^{x^2}3x^2\ln 4$ u) $\dfrac{5}{\sqrt{1-25^2}}$ v) $(\sin x)^{x^2}(2x\ln\sin x+x^2\cot x)$

4. a) i) $4x+y-4=0$ ii) $x-4y+33=0$ b) i) $2x-y-6=0$ ii) $x+2y+2=0$ c) i) $4x-5y+12=0$ ii) $5x+2y-13=0$

d) i) $y=x$ ii) $x+y-\pi=0$ e) i) $y=e$ ii) x = 1

5. $(\pi/4, 2/\sqrt{2})(5\pi/4, -2/\sqrt{2})$

Part B: Distance, Velocity & Acceleration

1. a) $t=2.5$ b) $t=5$ c) $a(t)=h''(t)=-9.8$ 2. a) $(a,b)(e,f)$ b) $(b,c)(f,\infty)$ c) $(-\infty,a)(d,e)$ d) (c,d)
3. a) 3456 b) 1450.24

Part C: Optimization

1. At 64 minutes 2. $20cm$ by $20cm$ by $10cm$ 3. 4 m by 4 m by 1 m $V=16\text{ m}^3$
4. a) $100m$ by 200m b) $50m$ by 300m 5. $\pi/2$

Part D: Maximum & Minimum Values

1. a) min $f(0)=0$ $f(1)=0$, max $f(-2)=36$ b) min $f(\pi/3)=-1/4$ $f(5\pi/3)=-1/4$, max $f(\pi)=2$
2. no turning points min $(0,-7)$ max $(2, 17/5)$

Part E: Curve Sketching

1. a) V.A. none H.A. none b) V.A. $\lim_{x\to -3^-} f(x)=-\infty$ $\lim_{x\to -3^+} f(x)=+\infty$ H.A. $y=0$ As $x\to+\infty$ $y\to 0$ from top, $x\to-\infty$ bottom

c) V.A. $\lim_{x\to -3^-} f(x)=-\infty$ $\lim_{x\to -3^+} f(x)=+\infty$ $\lim_{x\to -4^-} f(x)=-\infty$ $\lim_{x\to -4^+} f(x)=+\infty$ H.A. none

2. a) concave down $x<-4/5$, concave up $x>-4/5$ P. of I. $(-4/5, 238/25)$ b) concave down $x<-5/2$, concave up $x>-5/2$ P. of I. $(-5/2, 4672/125)$ 3. a) max $(1,4)$ min $(3,0)$ b) min $(1/2,-1/4)$

Forces and Velocity

1. An object is being towed by two ropes. The direction of forces of the ropes are N20' W and N 30° E. If the resultant force is 1000 N due north, find the magnitude of the tensions of each rope.

2. An object of mass 2 kg is hung by a rope attached to the ceiling. The object is held at equilibrium by a horizontal force. The rope makes an angle of 30° with the vertical. Find the magnitude of horizontal force and the tension of the rope.

3. An object of mass 2 kg rests on a ramp which is inclined at 30° to the horizontal. There is a frictional force of 5 N which exerts on the object up the ramp. Find the force that must be applied 20° to the ramp to keep the object at rest.

4. A man can swim at 3 m/s in still water. The current of a river is l m/s. If the man wishes to land at a point directly opposite to his present position, in what direction should he head?

Applications of Dot Product and Cross Product

1. Find the projection of $\vec{u}$ =(1,3,5) on $\vec{v}$ =(-1,3,-2)

2. ABCD is a// gram with A(-1,3,5), B(2, 3, 4), D(0,3,0). Find the area of the parallelogram ABCD using cross product.

3. A 5 N force which is along the direction of the vector (2, 3), moves an object from A (1, 3) to B(3, 7). Find the work done. The distance are in meters.

Vector, Parametric and Symmetric Equations of a Line in 3-space

Find the vector equation, parametric equation and, if possible, the symmetric equation of the line passing P(3, 1, 5)

1. with direction vector (1, 3, -7)
2. passing the point (5, 3, 4)
3. parallel to the line joining (5,4,3)and (-4,1,0)
4. parallel to $\frac{x-4}{3}=\frac{y+4}{3}=\frac{z-7}{5}$

Directions of a Line in 3-Space

1. Find the direction cosines and direction angles of the line $\frac{x-3}{1}=\frac{y-2}{2}=\frac{z-1}{3}$

2. A line through origin makes equal angles with all the three axis. Find the direction cosines of the line.

3. A line makes angles of 60° and 45° with the x-axis and y-axis respectively. What angle does it make with the positive z-axis?

Distance from a Point to a Line in 3-Space

1. Find the distance form the point P(2, 1, 2) to the line $\frac{x-1}{3}=\frac{y-3}{2}=\frac{z-1}{2}$

2. Given point P(0,1,5), and a line ℓ: $\frac{x}{1}=\frac{y}{2}=\frac{z-1}{1}$ find

a) the foot of perpendicular from the point P to the line ℓ
b) the line passing the point P and perpendicular to and cut the given line ℓ.
c) the distance from the point P to the line ℓ

Intersection of planes

1. Determine the intersection, if any, of each of the following sets of planes.

a)
$$4x+y+3z=-1$$
$$3y-2z=2$$
$$8x+5y+4z=5$$

b)
$$2x+y-3z=5$$
$$x-4y+z=4$$
$$3x-3y-2z=5$$

c)
$$3x-4y+z=-1$$
$$3x+7y+4z=38$$
$$6x-3y-z=7$$

Cawthra Park Secondary School
MCV 4U0 Final Examination

Teacher: Mr. V. Madian Date:

Name: ______________________________ Mark:______

INSTRUCTIONS:

1. **Calculators are permitted.**
2. **Answer all questions on the examination pages.**
3. **Exact answers are required for all questions unless specified.**

Part A Short Answer

1. A rubber ball is acted on by three forces. $\vec{F}_1$ has a magnitude of 10N and acts N30° E, $\vec{F}_2$ has a magnitude of 20N and acts S60°E and $\vec{F}_3$ has a magnitude of 30N and acts N10°E. Find the magnitude of the resultant after resolving the forces into components and determine the direction that the ball will move. (two decimals)

2. Determine if the vectors $(-1,0,1)$, $(2,1,1)$ and $(3,1,1)$ form a basis for R^3. If they do determine the co-ordinates of $(1,2,3)$ relative to the basis.

3. Determine the volume of the parallelepiped determined by the vectors $(2,1,1)$, $(2,3,2)$, and $(1,3,-2)$.

4. Determine the distance from $P(-2,1,3)$ to the line through $X(2,1,0)$ and $Y(1,1,1)$.

5. Determine where the line from $A(5,1,3)$ drawn perpendicular to the plane $3x+4y-5z=10$ hits the plane.

6. Two planes are given by $\vec{r}_1=(1,1,1)+s(2,1,0)+t(0,1,0),\ s,t\in R$ and $\vec{r}_2=(-1,1,3)+u(-1,1,0)+v(2,1,1),\ u,v\in R$. Determine the acute angle between these planes. (two decimals)

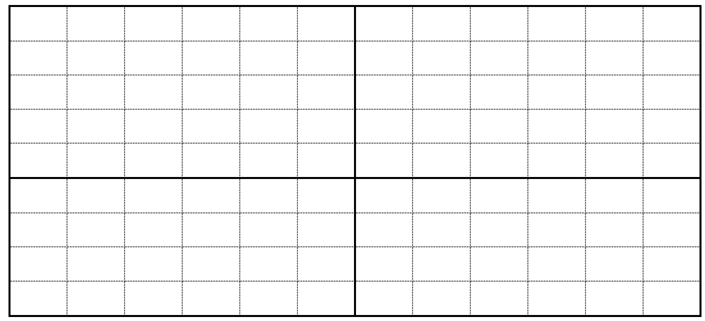
7. Determine the limit, if it exists:

a) $\lim\limits_{x\to 4}\frac{4-x}{x^2-16}$ b) $\lim\limits_{x\to 0}\frac{\sqrt{4-3x}-2}{x}$

c) $\lim\limits_{x\to 8}\frac{x-8}{\sqrt[3]{x}-2}$

8. Given $f(x)=\begin{cases}x^2, & if\ x\geq 1\\ -x, & if\ x<1\end{cases}$,

a) Sketch the graph of $f(x)$.



b) $\lim\limits_{x\to 2} f(x)=$______ c) $\lim\limits_{x\to -3} f(x)=$______ d) $\lim\limits_{x\to 1} f(x)=$______

9. The displacement, in metres, of a particle moving back and forth in a straight line is given by $s=2t^2-11t+15$, where t is measured in seconds.

a) Find the average velocity when $1\leq t\leq 2$.

b) Find the instantaneous velocity at $t=1$.

10. Use the definition of the derivative to find the derivative of $y=3x-x^2$.

11. Determine $\frac{dy}{dx}$ for each of the following:

a) $y=3x^5-\frac{5}{x}+2\sqrt[3]{x^2}+5$ b) $y=\left(3t^2-4t\right)^5$

c) $y=\sqrt{2x^2-7}\,(2x+5)^3$ ***Don't simplify question 2c).

12. Find the points on the graph of $y=\frac{9x}{x-3}$ where the slope of the tangent line is parallel to the line $27x+36y-13=0$.

13. If $y=\frac{1}{u}-u$ and $u=\frac{x+1}{x-2}$, find $\frac{dy}{dx}$ when $x=3$.

14. Find the equation of the normal to the curve $xy+4y^3=3x$ at the point $(2,1)$.

15. Starting at time $t=0$, a dragster accelerates down a deserted road, brakes and then, comes to a stop. The dragster's position function, $s(t)$, where s is in metres and t is in seconds, is given by $s(t)=6t^2-\frac{1}{5}t^3$.

a) How long does it take the dragster to stop?

b) How far has the dragster traveled at this time?

c) When does the dragster apply the brakes?

16. Find $\frac{dy}{dx}$ for each of the following:

a) $y=\left(e^x+e^{2x}\right)^4$

b) $y=\log_4\left(x^2+3\right)$

c) $y=In\left(\frac{e^{2x}-1}{e^{2x}+1}\right)$

d) f(x) = sec 5x tan 2x

2. If $In(xy)=x-y$, find $\frac{dy}{dx}$ at the point $(2,1)$.

17. Use logarithmic differentiation to find $y=\frac{\sqrt[5]{10x^2-3}}{e^{3x^2-4}}$ at the point $x=1$.

Round your final answer to the nearest tenth.

18. Find an expression for dy/dx for the curve 3xy - y2 = cos (2x2 + 3y)