MCR3U: Functions

Applications of Trigonometry

Part 1: 2D Scenarios

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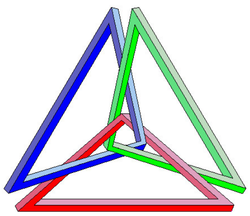
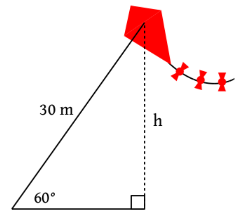


Applications of Primary Ratios

Example

A kite is flown at an angle of 60° to the ground, at the end of a 30 m string. Determine its height above the ground.

A sketch of the scenario shows a right triangle with a hypotenuse of 30 m and a height of h m.



Applications of Primary Ratios

Use the sine ratio to solve for h.

$$\begin{aligned}
\sin 60^\circ &= \frac{h}{30} \\
h &= 30 \sin 60^\circ \\
&= 30 \times \frac{\sqrt{3}}{2} \\
&= 15\sqrt{3} \\
&\approx 26
\end{aligned}$$

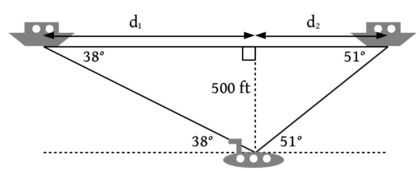
Thus, the kite is approximately 26 m above the ground.

Applications of Primary Ratios

Example

A submarine, 500 feet below the surface of the water, is directly between two ships. If the angles of elevation from the submarine to each ship are 38° and 51°, how far apart are the ships?

A picture of the scenario is shown, where d_1 and d_2 are the horizontal distances from each ship to the submarine.



Applications of Primary Ratios

Use the tangent ratio in each right triangle.

$$\begin{aligned}
\tan 38^\circ &= \frac{500}{d_1} & \tan 51^\circ &= \frac{500}{d_2} \\
d_1 &= \frac{500}{\tan 38^\circ} & d_2 &= \frac{500}{\tan 51^\circ} \\
d_1 &\approx 640 & d_2 &\approx 405
\end{aligned}$$

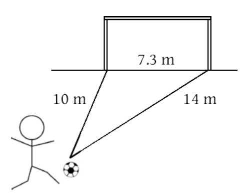
Therefore, the distance between the two ships is approximately $640 + 405 \approx 1\,045$ ft.

Applications of Sine and Cosine Laws

Example

A soccer player takes a shot on a standard net that is 7.3 m wide. If the player is 10 m from one goalpost and 14 from the other, through what angle can a goal be made?

Sketch a diagram as shown.



Applications of Sine and Cosine Laws

Since three side lengths are known, use the Cosine Law to find the angle.

$$\begin{aligned} 7.3^2 &= 10^2 + 14^2 - 2(10)(14)\cos\theta \\ \theta &= \cos^{-1}\left(\frac{7.3^2 - 10^2 - 14^2}{-2(10)(14)}\right) \\ &\approx 29.9^\circ \end{aligned}$$

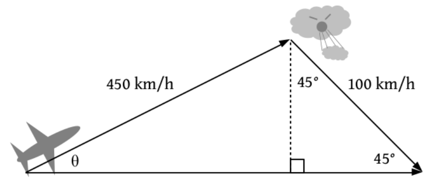
So, the player must shoot the ball through a 30° angle to score a goal.

Applications of Sine and Cosine Laws

Example

A pilot wishes to fly an airplane due East, but a strong wind blowing Southeast at 100 km/h keeps blowing the airplane off-course. If the airplane has a cruising speed of 450 km/h, in what direction should the pilot fly to reach the destination?

A diagram, showing the desired angle θ, is below.



Applications of Sine and Cosine Laws

In the previous diagram, information was given in an Angle-Side-Side format (45°, 100 km, 450 km).

Given the directions specified, only one triangle need be considered.

Use the Sine Law to determine the acute angle θ.

$$\begin{aligned} \frac{\sin\theta}{100} &= \frac{\sin 45^\circ}{450} \\ \sin\theta &= \frac{100\sin 45^\circ}{450} \\ \theta &= \sin^{-1}\left(\frac{\sqrt{2}}{9}\right) \\ &\approx 9^\circ \end{aligned}$$

The pilot should fly approximately 9° North of East.

Questions?