MCR3U: Functions

Transformations of Exponential Functions

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Transformations of Exponential Functions

While exponential functions often have the form $f(x) = a \cdot b^x$, they can also have additional transformations applied to them.

Recall that a function, $f(x)$, may be transformed to a related function, $g(x)$ with the form $g(x) = af(b(x-c)) + d$.

- a is a vertical stretch/compression, and possibly a vertical reflection (in the x-axis).
- b is a horizontal stretch/compression, and possibly a horizontal reflection (in the y-axis).
- c is a horizontal translation.
- d is a vertical translation.

The b and c values, being "inside" of the function, will appear in the exponent, such as in $f(x) = a \cdot 5^{b(x-c)} + d$ where the base function is $y = 5^x$.

Transformations of Exponential Functions

Example

State the transformations applied to $f(x) = 2^x$ to produce $g(x) = 3 \cdot 2^{x-4} - 1$, and sketch a graph.

- There is a vertical stretch by a factor of 3
- There is a horizontal translation 4 units right
- There is a vertical translation 1 unit down

The base function is $y = 2^x$.

The horizontal asymptote moves to $y = -1$.

The $f(x)$-intercept moves from $(0, 1)$ to $(0, 3)$ after the VS, then to $(4, 2)$ after the VT and HT.

The $g(x)$-intercept is $g(0) = 3 \cdot 2^{0-4} - 1 \approx -0.8$.

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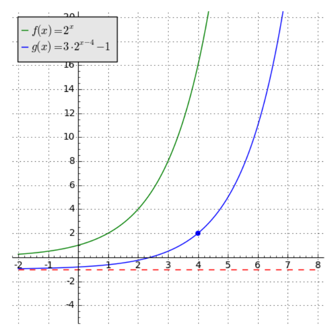

Transformations of Exponential Functions

Example

$f(x) = 3^x$ has the following transformations applied to it to produce $g(x)$:

- a vertical compression by a factor of $\frac{2}{3}$,
- a reflection in the x-axis,
- a horizontal stretch by a factor of 4,
- a horizontal translation 2 units left

Determine an equation for $g(x)$ and sketch a graph.

The base function is $y = 3^x$, and the parameters are $a = -\frac{2}{3}$, $b = \frac{1}{4}$ and $c = -2$.

Therefore, an equation is $g(x) = -\frac{2}{3} \cdot 3^{\frac{1}{4}(x+2)}$.

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The horizontal asymptote remains at $y = 0$.

The $f(x)$-intercept moves from $(0, 1)$ to $\left(0, -\frac{2}{3}\right)$ after the VC and VR, then to $\left(-2, -\frac{2}{3}\right)$ after the HT.

The $g(x)$-intercept is $g(0) = -\frac{2}{3} \cdot 3^{\frac{1}{4}(0+2)} \approx -1.15$.

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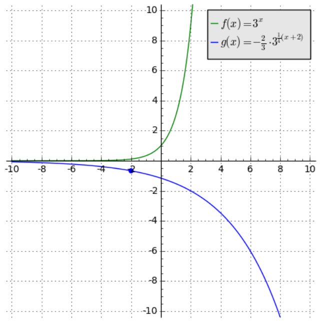
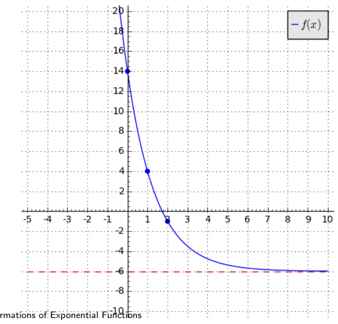


Transformations of Exponential Functions

Example

Determine an equation for the exponential function below.



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The horizontal asymptote is at $y = -6$, so all points on the graph have been translated down 6 units.

This includes the $f(x)$-intercept, which is at 14. Therefore, the initial value, a, should be $14 + 6 = 20$.

Since three successive points are $(0, 14)$, $(1, 4)$ and $(2, -1)$, the base b should be the ratio of their finite differences.

x	y	$\Delta 1$
0	14	
1	4	-10
2	-1	-5

Thus, $b = \frac{-5}{-10} = \frac{1}{2}$, and the equation is $f(x) = 20\left(\frac{1}{2}\right)^x - 6$.

Questions?