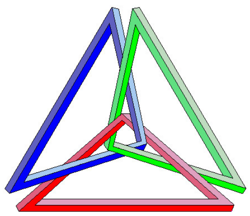
MCR3U: Functions

Primary and Secondary Trigonometric Ratios

J. Garvin

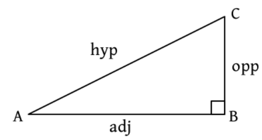


Primary Trigonometric Ratios

Recall the three *primary* trigonometric ratios for a right triangle.

Primary Trigonometric Ratios

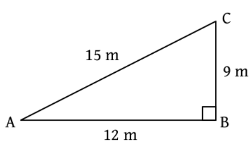
If ΔABC is a right triangle, such that $\angle A \neq 90^\circ$, then $\sin A = \dfrac{\text{opp}}{\text{hyp}}$, $\cos A = \dfrac{\text{adj}}{\text{hyp}}$, $\tan A = \dfrac{\text{opp}}{\text{adj}}$.



Primary Trigonometric Ratios

Example

Determine the three primary trigonometric ratios for $\angle A$.

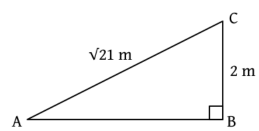


$\sin A = \frac{9}{15} = \frac{3}{5}$, $\cos A = \frac{12}{15} = \frac{4}{5}$, $\tan A = \frac{9}{12} = \frac{3}{4}$.

Primary Trigonometric Ratios

Example

Determine the three primary trigonometric ratios for $\angle A$.



Since we do not know the length of the adjacent side, use the Pythagorean Theorem to find it.

Primary Trigonometric Ratios

$$\begin{aligned} \sqrt{21}^2 &= 2^2 + \text{adj}^2 \\ 21 - 4 &= \text{adj}^2 \\ 17 &= \text{adj}^2 \\ \text{adj} &= \sqrt{17} \end{aligned}$$

So, $\sin A = \frac{2}{\sqrt{21}}$, $\cos A = \frac{\sqrt{17}}{\sqrt{21}}$, $\tan A = \frac{2}{\sqrt{17}}$.

Rationalizing Denominators

In the last example, all ratios contained radicals in their denominators.

It is common practice in mathematics to rewrite rational numbers so that any radicals appear in their numerators.

A simple way to do this is to note that a rational value $\frac{k}{\sqrt{x}}$ is equivalent to $\frac{k}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{k\sqrt{x}}{x}$.

This process is known as *rationalizing the denominator*.

Rationalizing Denominators

Example

Rationalize the ratios $\frac{2}{\sqrt{21}}$, $\frac{\sqrt{17}}{\sqrt{21}}$ and $\frac{2}{\sqrt{17}}$ from the previous example.

$$\frac{2}{\sqrt{21}} = \frac{2\sqrt{21}}{\sqrt{21}\sqrt{21}} = \frac{2\sqrt{21}}{21} \qquad \frac{\sqrt{17}}{\sqrt{21}} = \frac{\sqrt{17}\sqrt{21}}{\sqrt{21}\sqrt{21}} = \frac{\sqrt{357}}{21} \qquad \frac{2}{\sqrt{17}} = \frac{2\sqrt{17}}{\sqrt{17}\sqrt{17}} = \frac{2\sqrt{17}}{17}$$

Secondary Trigonometric Ratios

In addition to the primary ratios, there are three *secondary* ratios that are used less-frequently.

These ratios are *secant*, *cosecant* and *cotangent*.

Secondary Trigonometric Ratios

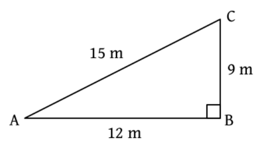
If ΔABC is a right triangle, such that $\angle A \neq 90°$, then $\sec A = \frac{\text{hyp}}{\text{adj}}$, $\csc A = \frac{\text{hyp}}{\text{opp}}$, $\cot A = \frac{\text{adj}}{\text{opp}}$.

The secondary ratios are sometimes called the *reciprocal* ratios, since $\csc A = \frac{1}{\sin A}$, $\sec A = \frac{1}{\cos A}$ and $\cot A = \frac{1}{\tan A}$.

Secondary Trigonometric Ratios

Example

Determine the three secondary trigonometric ratios for $\angle A$.



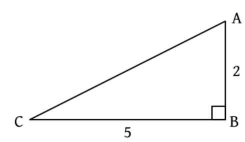
$\csc A = \frac{15}{9} = \frac{5}{3}$, $\sec A = \frac{15}{12} = \frac{5}{4}$, $\cot A = \frac{12}{9} = \frac{4}{3}$.

Secondary Trigonometric Ratios

Example

In ΔABC, $\cot A = \frac{2}{5}$. Determine the values of the other five ratios.

Draw a picture using the given information.



Use the Pythagorean Theorem to determine the length of the hypotenuse.

Primary Trigonometric Ratios

$$\begin{aligned} \text{hyp}^2 &= 2^2 + 5^2 \\ \text{hyp}^2 &= 29 \\ \text{hyp} &= \sqrt{29} \end{aligned}$$

So, $\sin A = \frac{5\sqrt{29}}{29}$, $\cos A = \frac{2\sqrt{29}}{29}$, $\tan A = \frac{5}{2}$, $\sec A = \frac{\sqrt{29}}{2}$ and $\csc A = \frac{\sqrt{29}}{5}$.

Questions?