

MCR3U: Functions

Multiplying and Dividing Rational Expressions

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Slide 1/10

Simplifying Rational Expressions

Recap

Simplify $\frac{2x^2 - 13x + 15}{25 - x^2}$.

$$\begin{aligned}\frac{2x^2 - 13x + 15}{25 - x^2} &= \frac{(2x - 3)(x - 5)}{(5 - x)(5 + x)} \\ &= \frac{(2x - 3)(x - 5)}{(x - 5)(x + 5)} \\ &= \frac{2x - 3}{x + 5}, x \neq 5, x \neq -5\end{aligned}$$

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Slide 2/10

Multiplying and Dividing Rational Expressions

Like rational numbers, rational expressions can be multiplied or divided using the same rules.

When multiplying rational expressions, simplify first. Then multiply the numerators, and multiply the denominators.

When dividing rational expressions, multiply the first rational expression by the reciprocal of the second.

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Slide 3/10

Multiplying Rational Expressions

Example

Simplify $\frac{x + 4}{x - 3} \cdot \frac{x - 3}{x - 2}$.

$$\begin{aligned}\frac{x + 4}{x - 3} \cdot \frac{x - 3}{x - 2} &= \frac{x + 4}{\cancel{x - 3}} \cdot \frac{\cancel{x - 3}}{x - 2} \\ &= \frac{x + 4}{x - 2}, x \neq 2, x \neq 3\end{aligned}$$

As always, state *all* restrictions on the variable.

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Slide 4/10

Multiplying Rational Expressions

Example

Simplify $\frac{x^2 - 2x - 3}{x + 2} \cdot \frac{x^2 + 7x + 10}{x + 1}$.

$$\begin{aligned}\frac{x^2 - 2x - 3}{x + 2} \cdot \frac{x^2 + 7x + 10}{x + 1} &= \frac{(x + 1)(x - 3)}{x + 2} \cdot \frac{(x + 2)(x + 5)}{x + 1} \\ &= (x - 3)(x + 5), x \neq -1, x \neq -2\end{aligned}$$

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Slide 5/10

Multiplying Rational Expressions

Your Turn

Simplify $\frac{49x^2 + 28x + 4}{7x^2 - 5x - 2} \cdot \frac{x^2 - 1}{x - 4}$.

$$\begin{aligned}\frac{49x^2 + 28x + 4}{7x^2 - 5x - 2} \cdot \frac{x^2 - 1}{x - 4} &= \frac{(7x + 2)^2}{(7x + 2)(x - 1)} \cdot \frac{(x - 1)(x + 1)}{x - 4} \\ &= \frac{(7x + 2)(x + 1)}{x - 4} \\ &= \frac{7x^2 + 9x + 2}{x - 4}, x \neq -\frac{2}{7}, x \neq 1, x \neq 4\end{aligned}$$

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Slide 6/10

Dividing Rational Expressions

Example

$$\text{Simplify } \frac{x-5}{x+4} \div \frac{x-5}{x+1}.$$

$$\begin{aligned} \frac{x-5}{x+4} \div \frac{x-5}{x+1} &= \frac{x-5}{x+4} \cdot \frac{x+1}{x-5} \\ &= \frac{x+1}{x+4}, x \neq -4, x \neq -1, x \neq 5 \end{aligned}$$

Note that restrictions occur in both denominators, and in the numerator of the second rational expression.

Dividing Rational Expressions

Example

$$\text{Simplify } \frac{x^2-16}{x^2+8x+16} \div \frac{4x}{x^2+4x}.$$

$$\begin{aligned} \frac{x^2-16}{x^2+8x+16} \div \frac{4x}{x^2+4x} &= \frac{(x+4)(x-4)}{(x+4)^2} \div \frac{4x}{x(x+4)} \\ &= \frac{(x+4)(x-4)}{(x+4)^2} \cdot \frac{x(x+4)}{4x} \\ &= \frac{x-4}{4}, x \neq 0, x \neq -4 \end{aligned}$$

Dividing Rational Expressions

Your Turn

$$\text{Simplify } \frac{3-3x^2}{5x^2-15x-20} \div \frac{6x^2-12x}{10x+40}.$$

$$\begin{aligned} \frac{3-3x^2}{5x^2-15x-20} \div \frac{6x^2-12x}{10x+40} &= -\frac{3(x-1)(x+1)}{5(x+1)(x-4)} \div \frac{6x(x-2)}{10(x+4)} \\ &= -\frac{3(x-1)(x+1)}{5(x+1)(x-4)} \cdot \frac{10(x+4)}{6x(x-2)} \\ &= -\frac{(x-1)(x+4)}{x(x-2)(x-4)} \\ & x \neq -4, x \neq -1, x \neq 0, x \neq 2, x \neq 4 \end{aligned}$$

Questions?

