Mapping Diagrams and Function Notation

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Mapping Diagrams

A mapping diagram is a visual method of connecting elements in the domain of a relation to those in its range. Mapping diagrams are useful when the relation is specified as a set of ordered pairs.

If a mapping diagram maps one element in the domain to more than one element in the range, then the relation is not a function.

Example

Using the mapping diagram below, state the domain and range of the relation, and determine if it is a function.

Domain: \( \{1, 2, 3\} \) Range: \( \{4, 5, 6\} \)

The relation is a function, since each element in the domain maps to exactly one element in the range.

Example

Using the mapping diagram below, determine if the relation is a function.

Since the value of 1 in the domain maps to both 4 and 5 in the range, the relation is not a function.

Function Notation

Suppose you were writing a report, and had data involving the following three linear functions:

\[ y = 2x + 1 \]
\[ y = 7x + 5 \]
\[ y = \frac{1}{2}x + 4 \]
Function Notation

While each equation is distinct, it is difficult to refer to an individual function. For example, how would you indicate that the graph of $y = 2x + 5$ has a $y$-value of 9 when $x = 2$? You could refer to it as in the previous sentence, but it is wordy. You could refer to it by its $y$-intercept, but what if another function had the same one? You could mention that it is the green graph, but what if the report is published in black-and-white? A better solution is to use function notation to differentiate each function from another.

For example, the notation $f(x)$ is quite common, as are $g(x)$ and $h(x)$, but other variables can be used as well.

- $A(r) = \pi r^2$ describes the area of a circle, based on its radius $r$.
- $V(e) = e^3$ describes the volume of a cube based on an edge length $e$.
- $d(t) = 4.9t^2$ describes the distance travelled by a falling object after $t$ seconds.

Returning to the example of the linear relations, it is easy to refer to individual functions if they are labelled $f(x) = 2x + 5$, $g(x) = 2x + 1$ and $h(x) = \frac{1}{2}x + 4$.

To indicate that $y = 2x + 5$ has a $y$-value of 9 when $x = 2$, we would simply say $f(2) = 9$.

In this case, the $x$ in $f(x)$ has been replaced by 2 to give $f(2)$.

This indicates that anywhere in the function where the variable $x$ appears, we substitute a value of 2 in its place.

Example
Let $f(x) = x^2 - 5x + 1$. Determine $f(0)$, $f(3)$ and $f(-2)$.

\[
\begin{align*}
f(0) &= (0)^2 - 5(0) + 1 \\
&= 1 \\
f(3) &= (3)^2 - 5(3) + 1 \\
&= -5 \\
f(-2) &= (-2)^2 - 5(-2) + 1 \\
&= 15
\end{align*}
\]

Example
Let $g(x) = \sqrt{2x} - 5$. Solve for $x$ when $g(x) = 4$.

\[
\begin{align*}
g(x) &= \sqrt{2x} - 5 \\
4 &= \sqrt{2x} - 5 \\
16 &= 2x - 5 \\
21 &= 2x \\
x &= \frac{21}{2}
\end{align*}
\]

Example
Let $f(x) = 3x - 1$ and $g(x) = 5x + 3$. Determine the value of $f(x)$ such that $f(x) = g(x)$.

\[
\begin{align*}
f(x) &= g(x) \\
3x - 1 &= 5x + 3 \\
2x &= -4 \\
x &= -2 \\
f(-2) &= 3(-2) - 1 \\
f(-2) &= -7
\end{align*}
\]
Function Notation

Example

AN EXAMPLE WHERE $f(2x+1)$.

SOLUTION

The volume of copper in a cable with an inner radius of 2 cm depends on the length of the wire. Use function notation to define a function to describe the volume of copper, then determine:

- The volume when the length is 15 cm.
- The length when the volume is 500 cm$^3$.

The volume of a cylinder, given a radius $r$ and a length $l$, is $\pi r^2 l$.

Using function notation where $V$ is the volume, $l$ the length and $r = 2$, use $V(l) = 4\pi l$.

To determine the volume for a length of 15 cm, find $V(15)$.

$$V(15) = 4\pi (15) 
\approx 188.5 cm^3$$

To determine the length for a volume of 500 cm$^3$, find $l$ when $V(l) = 500$.

$$500 = 4\pi l 
\Rightarrow l = \frac{500}{4\pi} 
\approx 39.8 cm$$

A labelling system similar to function notation is mapping notation, which uses an arrow in place of an equals sign.

For example, the function $f : x \rightarrow x^2 - 4$ is read "function $f$ maps $x$ to $x^2 - 4" and is equivalent to $f(x) = x^2 - 4$.

We will typically use function notation in this course, but you should be familiar with both systems.

Questions?