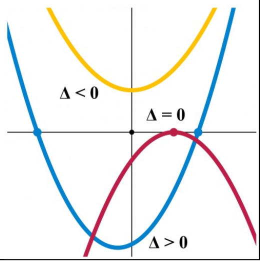
MCR3U: Functions

Equations of Quadratic Functions

J. Garvin



Forms of Quadratic Functions

Quadratic functions can be represented in three ways.

- Standard form, $f(x) = ax^2 + bx + c$
- Vertex form, $g(x) = a(x-h)^2 + k$
- Factored form, $h(x) = a(x-r)(x-s)$

All quadratic functions can be represented using standard form or vertex form, but it may not be possible to express a quadratic function using factored form.

Quadratic Functions in Vertex Form

Example

Determine the equation of the quadratic function with a vertex at $V(-2,3)$, that passes through $P(-4,1)$.

Use vertex form, since we know its coordinates.

$$\begin{aligned} f(x) &= a(x-h)^2 + k \\ 1 &= a(-4+2)^2 + 3 \\ 1 &= 4a + 3 \\ 4a &= -2 \\ a &= -\tfrac{1}{2} \end{aligned}$$

The equation is $f(x) = -\frac{1}{2}(x+2)^2 + 3$.

Quadratic Functions in Standard Form

Example

Determine the equation of the quadratic function with zeroes at 3 and -15 and a y-intercept of 9.

Use factored form, since we know the zeroes.

$$\begin{aligned} g(x) &= a(x-r)(x-s) \\ 9 &= a(0-3)(0+15) \\ -45a &= 9 \\ a &= -\frac{1}{5} \end{aligned}$$

The equation is $g(x) = -\frac{1}{5}(x-3)(x+15)$.

Quadratic Functions in Factored Form

Example

Determine the equation of the quadratic function with its only zero at -4 passing through $P(-7,18)$.

Since the quadratic has only one zero, it must be a perfect square.

$$\begin{aligned} f(x) &= a(x-h)^2 \\ 18 &= a(-7+4)^2 \\ 9a &= 18 \\ a &= 2 \end{aligned}$$

The equation is $f(x) = 2(x+4)^2$.

Quadratic Functions in Standard Form

It requires a bit more work to express a quadratic function in standard form.

It is generally a good idea to begin with either vertex or factored form, then expand to put the quadratic in standard form.

Since this is probably the least useful form of the three, it is not usually recommended to convert to standard form unless explicitly instructed to do so.

Quadratic Functions in Standard Form

Example

Convert $f(x) = -3(x-5)^2 + 1$ to standard form.

$$\begin{aligned} f(x) &= -3(x^2 - 10x + 25) + 1 \\ &= -3x^2 + 30x - 75 + 1 \\ &= -3x^2 + 30x - 74 \end{aligned}$$

Quadratic Functions in Standard Form

Example

Convert $g(x) = \frac{1}{4}(x-2)(x+8)$ to standard form.

$$\begin{aligned} g(x) &= \tfrac{1}{4}(x^2 + 6x - 16) \\ &= \tfrac{1}{4}x^2 + \tfrac{3}{2}x - 4 \end{aligned}$$

Quadratic Functions in Standard Form

Example

Determine the equation of the quadratic function with zeroes at $2-\sqrt{3}$ and $2+\sqrt{3}$, passing through $P(3,8)$, and express your answer in standard form.

Start with factored form.

$$\begin{aligned} g(x) &= a(x-r)(x-s) \\ &= a(x - [2-\sqrt{3}])(x - [2+\sqrt{3}]) \\ &= a([x-2] + \sqrt{3})([x-2] - \sqrt{3}) \\ &= a([x-2]^2 - 3) \\ &= a(x^2 - 4x + 4 - 3) \\ &= a(x^2 - 4x + 1) \end{aligned}$$

Quadratic Functions in Standard Form

Now, substitute in x and $g(x)$.

$$\begin{aligned} 8 &= a(3^2 - 4(3) + 1) \\ 8 &= -2a \\ a &= -4 \\ g(x) &= -4(x^2 - 4x + 1) \\ &= -4x^2 + 16x - 4 \end{aligned}$$

Quadratic Functions in Standard Form

Your Turn

Determine the equation of the quadratic function with zeroes at $5-\sqrt{2}$ and $5+\sqrt{2}$, passing through $P(-1,17)$, and express your answer in standard form.

$$\begin{aligned} h(x) &= a(x-r)(x-s) \\ &= a(x - [5-\sqrt{2}])(x - [5+\sqrt{2}]) \\ &= a([x-5] + \sqrt{2})([x-5] - \sqrt{2}) \\ &= a([x-5]^2 - 2) \\ &= a(x^2 - 10x + 23) \end{aligned}$$

Quadratic Functions in Standard Form

$$\begin{aligned} 17 &= a((-1)^2 - 10(-1) + 23) \\ 17 &= 34a \\ a &= \tfrac{1}{2} \\ h(x) &= \tfrac{1}{2}(x^2 - 10x + 23) \\ &= \tfrac{1}{2}x^2 - 5x + \tfrac{23}{2} \end{aligned}$$

Modelling with Quadratic Functions

Example

A parabolic arch is 30 m wide and supported, in part, by a horizontal crossbeam 5 m above the ground. At a distance of 3 m from the centre of the arch, the height is 9 m. Determine the length of the crossbar, and the maximum height of the arch.

First, determine an equation to model the height of the arch, $h(d)$ m, based on the distance, d m, from the centre.

$$\begin{aligned} h(d) &= a(d-r)(d-s) \\ 9 &= a(3-15)(3+15) \\ 9 &= -216a \\ a &= -\tfrac{1}{24} \\ h(d) &= -\tfrac{1}{24}(d-15)(d+15) \end{aligned}$$

Modelling with Quadratic Functions

The length of the crossbar can be found by substituting $h(d) = 5$.

$$\begin{aligned} 5 &= -\tfrac{1}{24}(d-15)(d+15) \\ -120 &= d^2 - 225 \\ d^2 &= 105 \\ d &= \pm\sqrt{105} \\ d &\approx \pm 10.25 \end{aligned}$$

The length of the crossbar is the difference of the distances, or $10.25 - (-10.25) \approx 20.5$ m.

Modelling with Quadratic Functions

Finding the maximum height of the arch is easy, since it is centred about the vertical axis.

Substitute $d = 0$ to find the maximum height.

$$\begin{aligned} h(0) &= -\tfrac{1}{24}(0-15)(0+15) \\ &= \tfrac{225}{24} \\ &= \tfrac{75}{8} \\ &= 9.375 \text{ m} \end{aligned}$$

The maximum height of the arch is 9.375 metres.

Questions?